華中科技大學
HUAZHONG UNIVERSITY OF SCIENCE AND TECHNOLOGY



Design of Hybrid Solid-Lattice Structures via Multi-material Topology Optimization Approach

XIA Liang【夏凉】

School of Mechanical Science and Engineering
Huazhong University of Science and Technology, Wuhan, China

2026-11-03

提纲
Outline

求明德厚学
是创新

华中科技大学
中国 武汉

1. **Overview & Motivation**
2. Multi-material interpolation models
3. Design method for hybrid structures
4. Applications: thermo-mech coupling & buckling
5. Conclusion

Lightweight hybrid “solid-lattice” structures

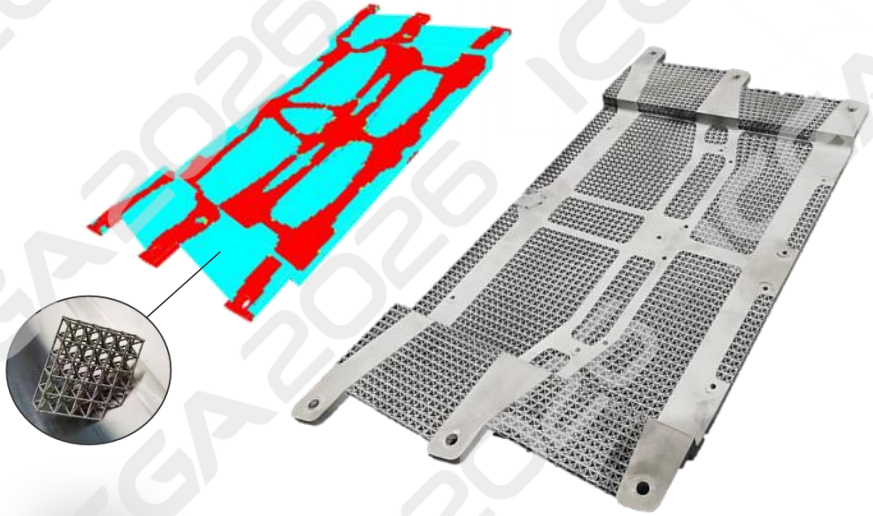
- The **“solid frame”** provides the necessary stiffness to withstand primary mechanical loads
- The **“core lattices”** enhance the overall structural stability against load variations, vibrations, buckling, impacts, and with potentially additional functionalities

PROMINENT POTENTIAL AEROSPACE APPLICATIONS



Gradient Lattice Structured Bracket
collaboratively by Altair, Materialise and Renishaw

www.altair.com/resource/form-and-function-metal-3d-printing-of-hybrid-lattice



Lightweight Equipment Mounting Plate
with solid material stiffeners and uniform lattice infilling

Li Y, Gao T, Zhang WH, Chinese Journal of Aeronautics, 2023

Brief overview: *single-material* approaches

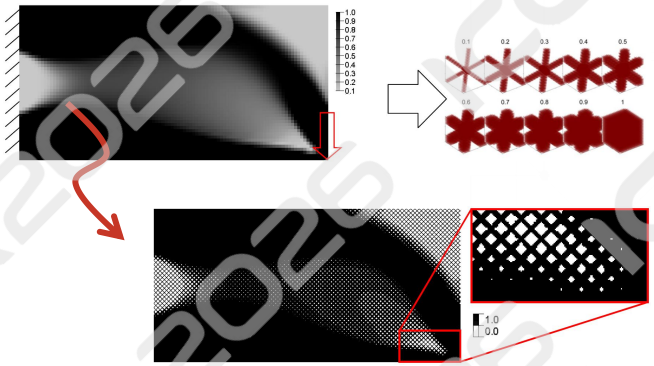
Homogenization Method (1988)

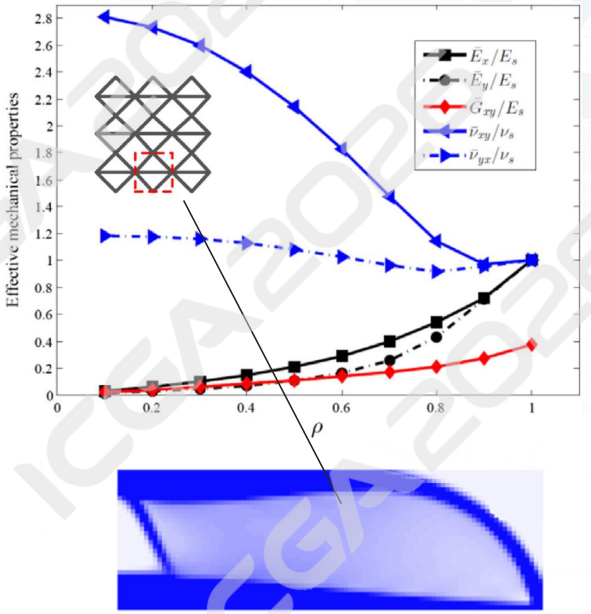
SIMP material interpolation model (1989)

Simplified **I**sotropic **M**aterial with **P**enalization

$$E = \rho^p E_0 \quad (p \geq 3)$$

↓ penalization-free ($p = 1$, 2011)

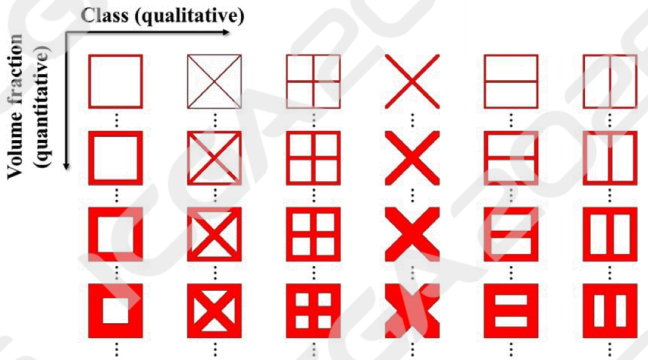


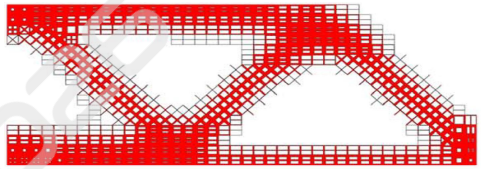


Effective Lattice Material Model (2015, 2017)

commercial software design flow

$$\bar{C}_{ijkl}^{\text{lattice}} = f^{(ijkl)}(\rho) C_{ijkl}^{\text{solid}}$$





Shape Morphing Material Model (2018, 2021)

with additional geometry morphing parameters

$$\bar{C}_{ijkl}^{\text{lattice}} = f^{(ijkl)}(\rho, z_1, z_2, \ldots)\ C_{ijkl}^{\text{solid}}$$

REMARKS: ① an integrated material description ② challenging to adjust solid and lattice volumes precisely

Bendsøe & Kikuchi 1988, Bendsøe 1989, Brackett et al. 2011, Zhang et al. 2015, Wang et al. 2017, Cheng et al. 2017, Wang et al. 2018, Wang et al. 2021

Brief overview: *multi-material* approaches

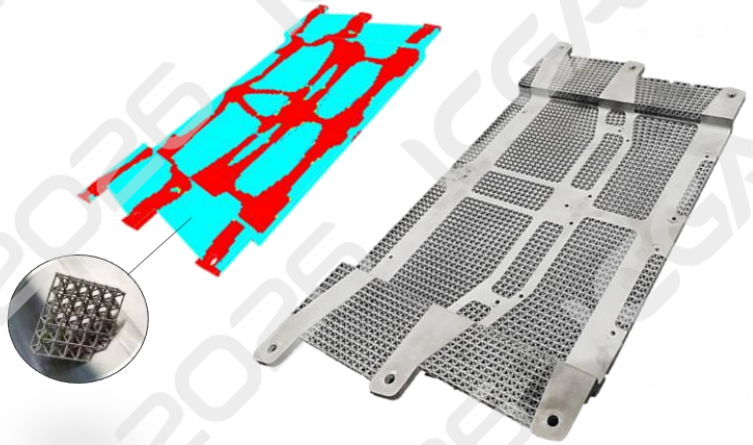
DMO multi-material Model (2005)

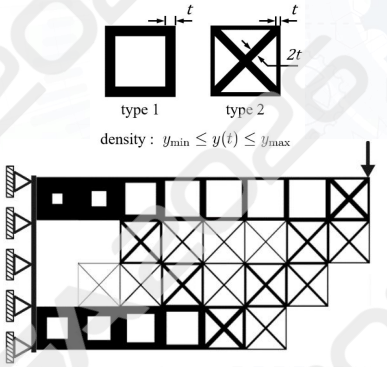
Discrete **M**aterial **O**ptimization / **UMMI** (2011)

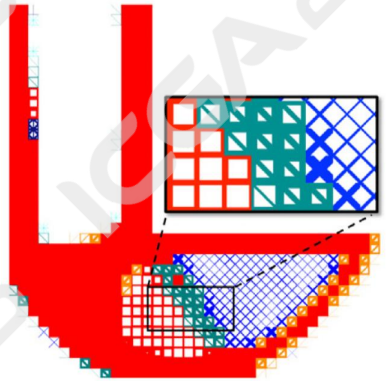
$$\mathbf{C} = \sum_{i=1}^{M} \left[\rho_i^p \prod_{j=1}^{M} \left(1 - \rho_{j\neq i}^p\right) \right] \mathbf{C}^{(i)}$$

expression for two solid phases and void

$$\mathbf{C} = \rho_1^p \left(1 - \rho_2^p\right)\mathbf{C}^{(1)} + \rho_2^p \left(1 - \rho_1^p\right)\mathbf{C}^{(2)}$$







REMARKS: ① both single- and multi-material models have proven capable of creating hybrid "solid-lattice" structures ② comparatively, multi-material model provides greater design flexibility and is potentially better suited for practical engineering applications

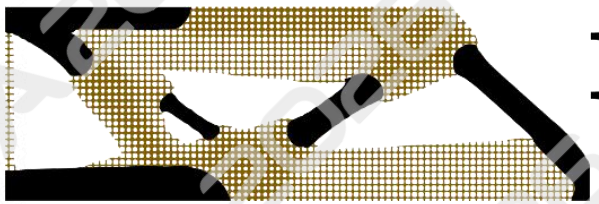
Motivation 1: explore the potential of multi-material models in creating hybrid "solid-lattice" structures

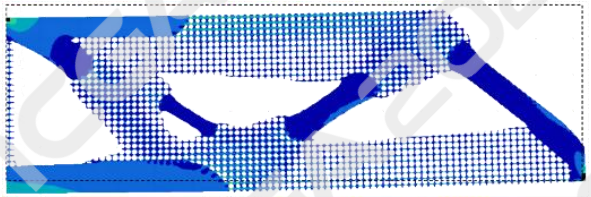
Motivation 2: verify the advantages of hybrid structural forms across diverse problem scenarios

Lund & Stegmann 2005, Gao & Zhang 2011, Liu & Xia 2020, Chan et al. 2022, Li, Gao, Zhang et al. 2023 ,Li, Gao & Xiao 2023, Chandrasekhar, Sridhara & Suresh 2023

Our recent progress on the subject *since 2023*

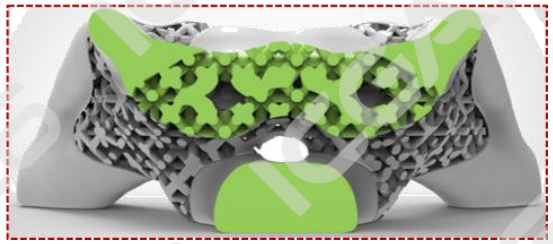




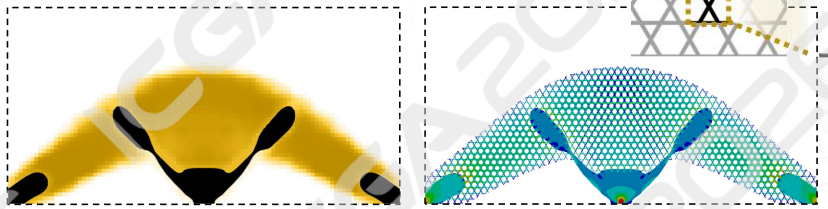


compliance

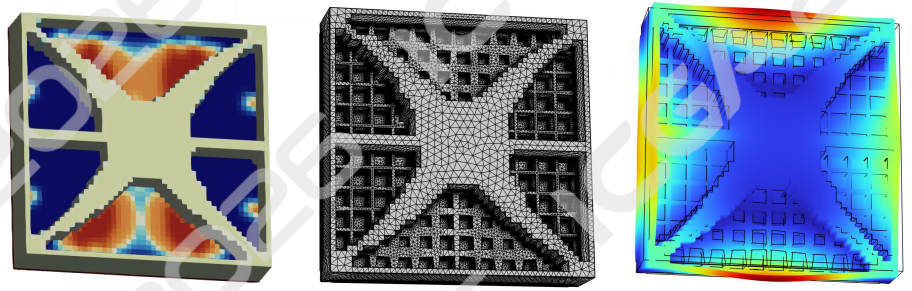




natural frequency



thermal-mechanical coupling



buckling resistance

1) A multi-material topology optimization approach to hybrid material structures with gradient lattices, **Comput Methods Appl Mech Eng**, **2024**, 425:116969
2) Hybrid material topology optimization of solid-lattice structures for natural frequency maximization, **Advances in Engineering Software**, **2025**, 208:103961
3) Thermal-mechanical design of hybrid solid-lattice structures using multi-material topology optimization, **Composite Structures**, **2026**, 379:119998
4) Hierarchical stiffened design of thin-walled structures using a density-based topology optimization method, **Thin-walled Structures**, submitted

提纲
Outline

求是创新 明德厚学

华中科技大学
中国 武汉

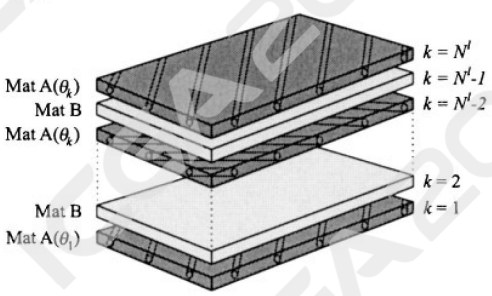
1. Overview & Motivation
2. **Multi-material interpolation models**
3. Design method for hybrid structures
4. Applications: thermo-mech coupling & buckling
5. Conclusion

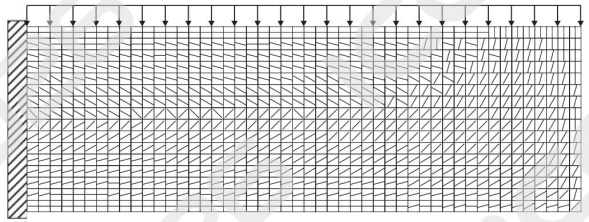
Main-stream multi-material interpolation models

Discrete Material Optimization model (2005)

Uniform Multiphase Materials Interpolation (UMMI, 2011)
expression for two solid phases and void

$$C_{ijkl} = \rho_1^p \left(1-\rho_2^p\right) C_{ijkl}^{(1)} + \rho_2^p \left(1-\rho_1^p\right) C_{ijkl}^{(2)}$$





12 fiber candidate directions
$[90°, \pm 75°, \pm 60°, \pm 45°, \pm 30°, \pm 15°, 0°]$

recursive SIMP model (2003)

Recursive Multiphase Materials Interpolation (RMMI, 2011)
expression for two solid phases and void

$$C_{ijkl} = \rho_1^p \left(\rho_2^p C_{ijkl}^{(1)} + \left(1-\rho_2^p\right) C_{ijkl}^{(2)}\right)$$



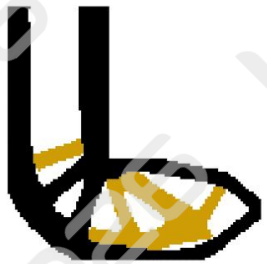
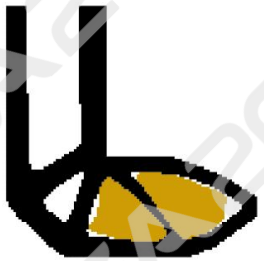
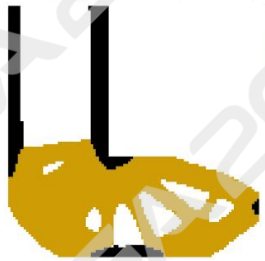
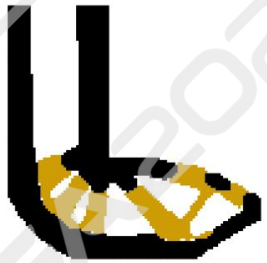
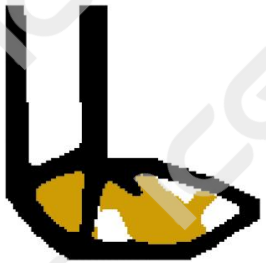
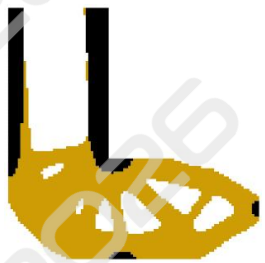
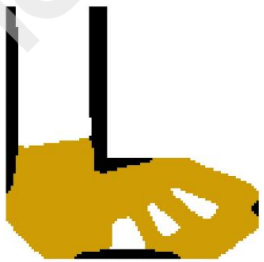
$C^2/C^1 = 0.01$ $C^2/C^1 = 0.2$ $C^2/C^1 = 0.5$

Recursive SIMP prioritizes the first material phase due to its dominant mechanical behavior over the second phase, making it potentially more suited for designing hybrid "solid-lattice" structures

Bendsøe & Sigmund 2003, Lund & Stegmann 2005, Gao & Zhang 2011

DMO *vs.* r-SIMP: a comparative demonstration

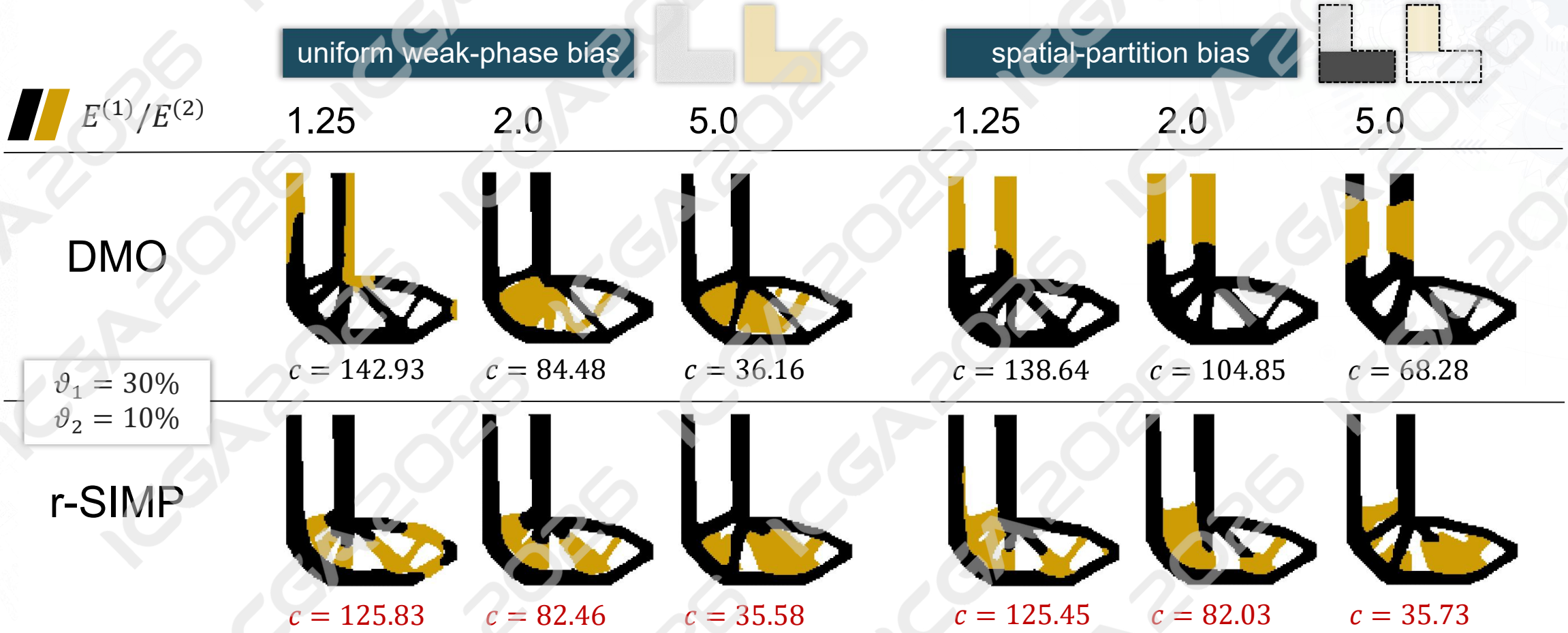
- **Material volume fraction**: effects on design results

	$\vartheta_1 = 30\%, \vartheta_2 = 10\%$			$\vartheta_1 = 10\%, \vartheta_2 = 30\%$		
$E^{(1)}/E^{(2)}$	1.25	2.0	5.0	1.25	2.0	5.0
DMO	 $c = 129.42$	 $c = 84.98$	 $c = 36.63$	 $c = 153.32$	 $c = 143.72$	 $c = 67.51$
r-SIMP	 $c = 125.69$	 $c = 82.35$	 $c = 35.52$	 $c = 136.46$	 $c = 105.93$	 $c = 62.59$

REMARKS: r-SIMP outperforms DMO in distributing the strong phase, yielding higher load-bearing capacity — and the advantage grows as the weak-phase volume fraction rises.

DMO *vs.* r-SIMP: a comparative demonstration

- **Initial variables setting**: effects on design results



REMARKS: under different initial variables settiings, r-SIMP produces more reasonable layouts and better load-bearing performance than DMO — and remains less sensitive to the initial design.

提纲
Outline

求是创新 明德厚学

华中科技大学
中国 武汉

1. Overview & Motivation
2. Multi-material interpolation models
3. **Design method for hybrid structures**
4. Applications: thermo-mech coupling & buckling
5. Conclusion

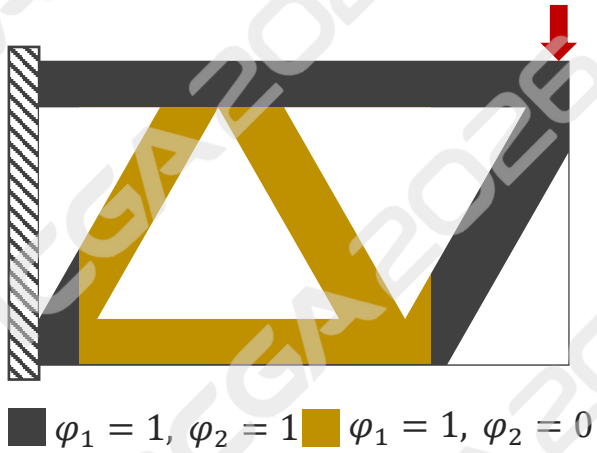
Hybrid material interpolation model

- Extension of the **recursive SIMP** multi-material model

$$C_{ijkl}(\varphi_1,\varphi_2)=\varphi_1^p\left(\varphi_2^p C_{ijkl}^{(1)}+\left(1-\varphi_2^p\right)C_{ijkl}^{(2)}\right)$$ recursive SIMP model

$$C_{ijkl}(\varphi_1,\varphi_2,\eta)=\varphi_1^p\left(\varphi_2^p C_{ijkl}^{(1)}+\left(1-\varphi_2^p\right)C_{ijkl}^{\mathrm{grad}}(\eta)\right)$$ gradient homogeneous material

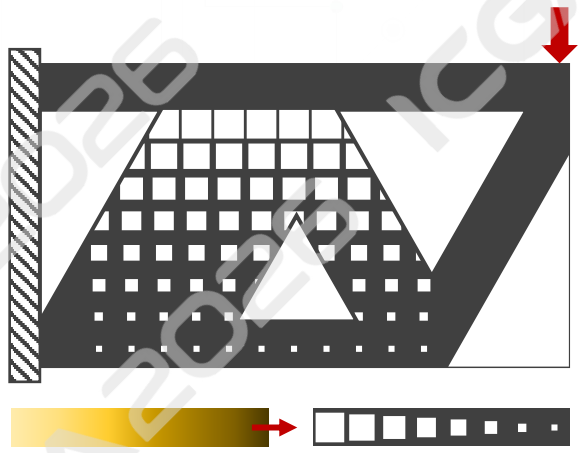
$$C_{ijkl}(\varphi_1,\varphi_2,\eta)=\varphi_1^p\left(\varphi_2^p C_{ijkl}^{(1)}+\left(1-\varphi_2^p\right)\overline{C}_{ijkl}^{\mathrm{lattice}}(\eta)\right)$$ spatially gradient lattices



Multi-material design
two solid material phases and void



Hybrid material design
solid and gradient phases, and void



Hybrid material design
solid and gradient latices, and void

Hybrid material interpolation model

- Extension of the **recursive SIMP** multi-material model

elasticity tensors: $C_{ijkl}(\varphi_1, \varphi_2, \eta) = \varphi_1^p \left(\varphi_2^p C_{ijkl}^{\text{solid}} + (1 - \varphi_2^p) \overline{C}_{ijkl}^{\text{lattice}}(\eta) \right)$

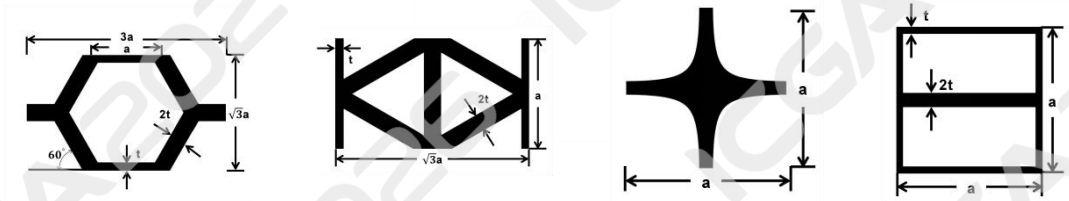
Numerical Homogenization

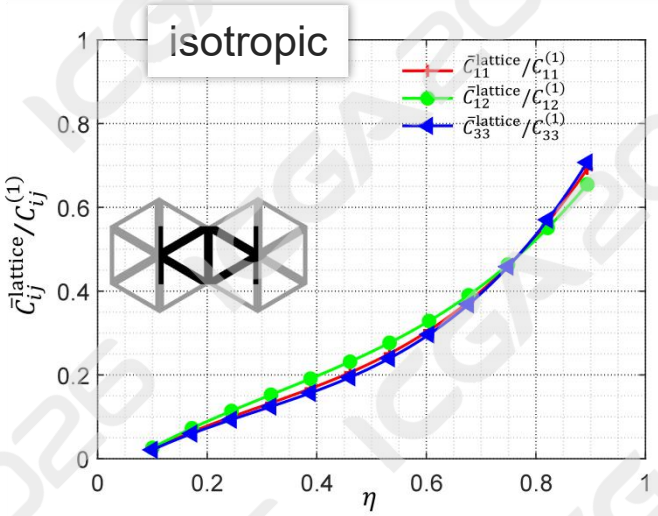
$$\overline{\sigma}_{ij} = \overline{C}_{ijkl}^{\text{lattice}}(\eta)\, \overline{\varepsilon}_{kl}$$

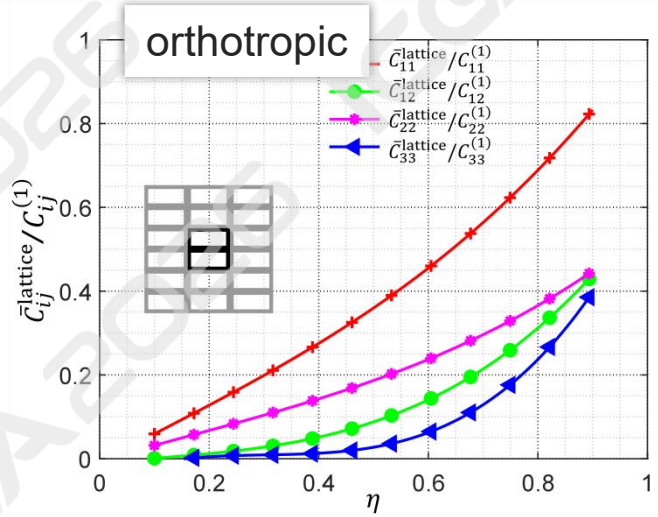
$$\overline{C}_{ijkl}^{\text{lattice}}(\eta) \approx f^{(ijkl)}(\eta) C_{ijkl}^{\text{solid}}$$

the lattice constituent material phase is the same as the primary solid material phase

$f^{(ijkl)}(\eta)$ are the interpolation fitting functions for each of the tensors







Hybrid “solid-lattice” topology optimization

$$\min : f_{\text{obj}} = \mathbf{U}^{\mathrm{T}}\mathbf{K}\mathbf{U}$$

$$\text{subject to} : \mathbf{K}\mathbf{U} = \mathbf{F}$$

$$: g_{\text{solid}}(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2) \leq 0$$

$$: g_{\text{lattice}}(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \boldsymbol{\eta}) \leq 0$$

$$: 0 \leq \varphi_{1,e} \leq 1 \, , \, e = 1, \, \ldots, \, N$$

$$: 0 \leq \varphi_{2,e} \leq 1 \, , \, e = 1, \, \ldots, \, N$$

$$: \eta_{\min} \leq \eta_e \leq \eta_{\max} \, , \, e = 1, \, \ldots, \, N$$



$$g_{\text{solid}}(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2) = \frac{V_{\text{solid}}(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2)}{\vartheta_{\text{solid}} V_{\text{domain}}} - 1 \leq 0$$

$$g_{\text{lattice}}(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \boldsymbol{\eta}) = \frac{V_{\text{lattice}}(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \boldsymbol{\eta})}{\vartheta_{\text{lattice}} V_{\text{domain}}} - 1 \leq 0$$

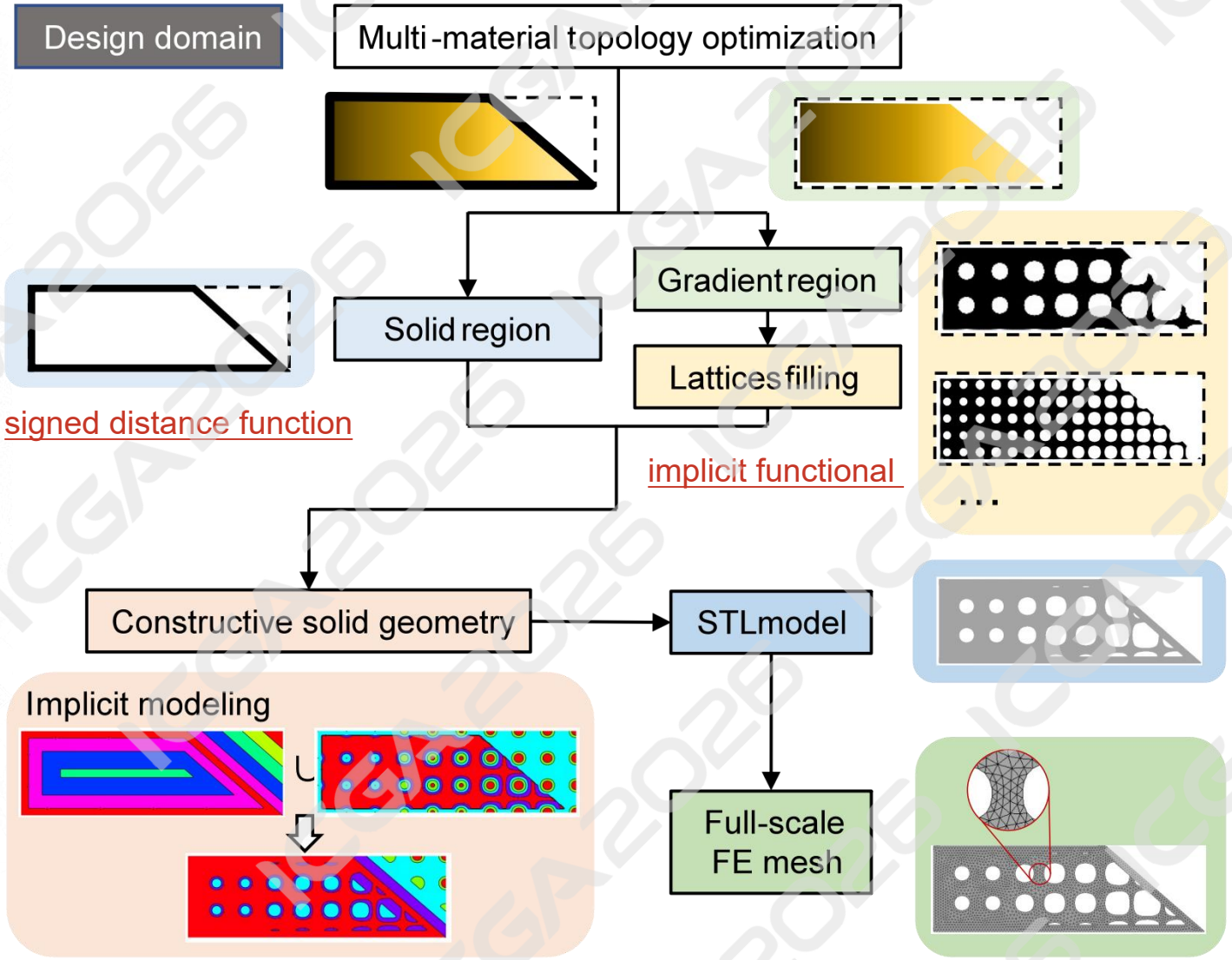
ϑ_{solid} : volume fraction of the primary solid material phase over the design domain

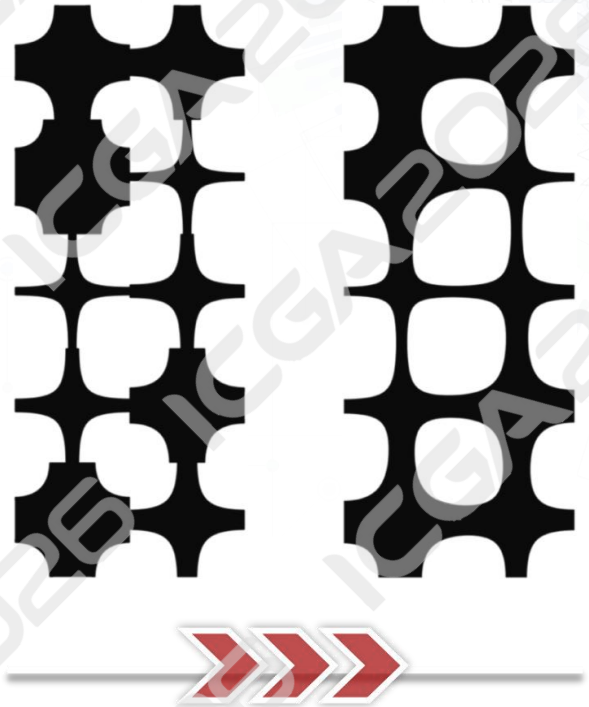
$\vartheta_{\text{lattice}}$: volume fraction of the lattices over the design domain

↑ **COMPLIANCE MINIMIZATION FORMULATION**

− adapts also to thermal-mechanical coupling, dynamic and other physical design scenarios

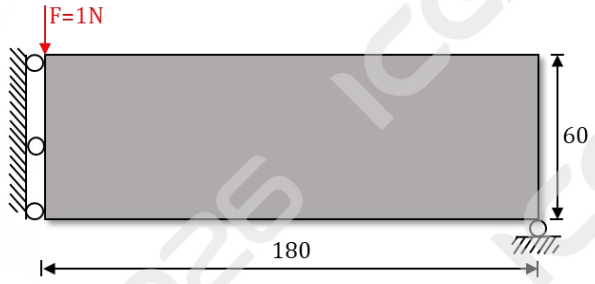
Post-processing field-driven lattice generation





bilinear interpolation scheme
refining the implicit representation functions of weakly connected lattices

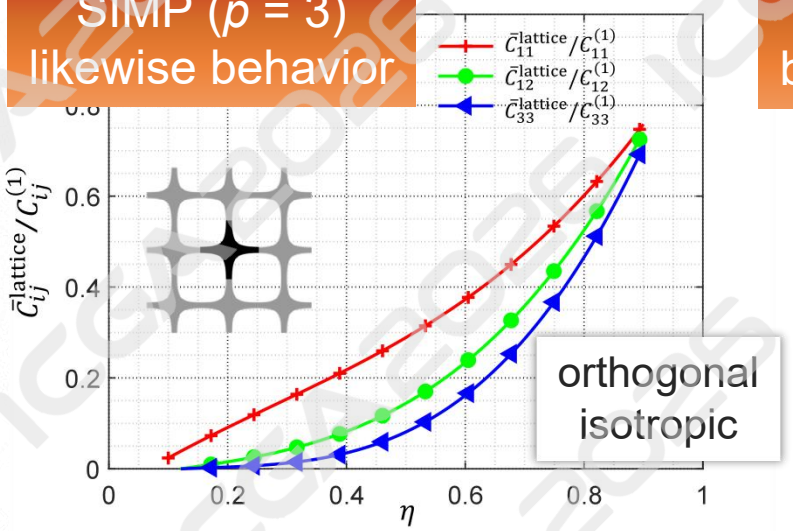
Benchmark half-MBB beam design (compliance Min.)



volume fraction settings:	$\vartheta_{\text{solid}} = 20\%,\ \vartheta_{\text{lattice}} = 20\%$
lattice density ranges:	$\eta_{\min} = 0.3\ ,\ \eta_{\max} = 0.7$

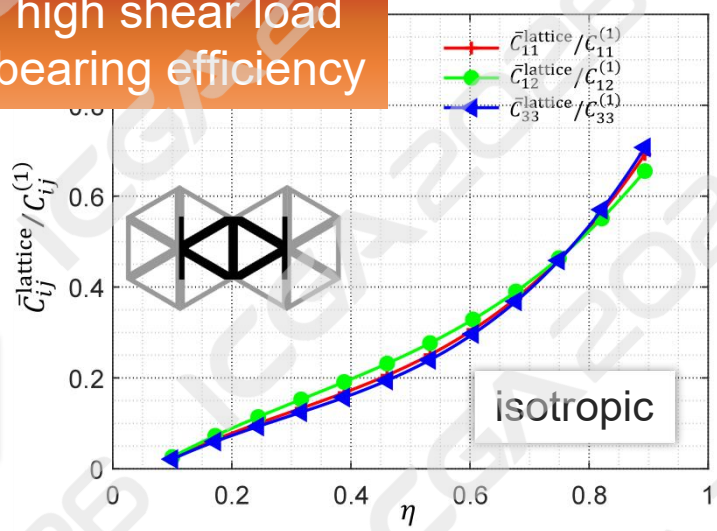


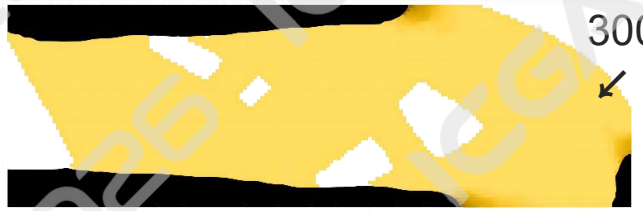
SIMP (p = 3) likewise behavior



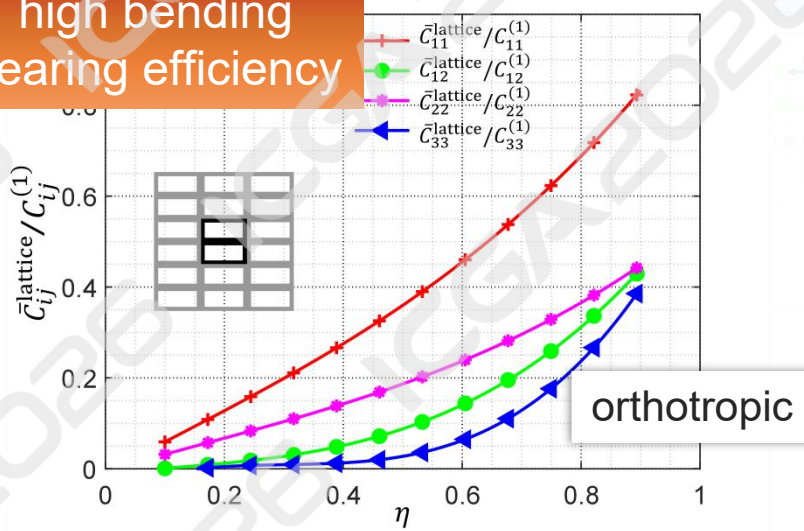


high shear load bearing efficiency





high bending bearing efficiency





REMARKS: ① hybrid material layouts are lattice-dependent ② low mechanical efficiency of inter-density lattices

Benchmark half-MBB beam design (compliance Min.)

volume fraction settings:	ϑ_{solid} / $\vartheta_{\text{lattice}}$
lattice density ranges:	$\eta_{\min} = 0.3$, $\eta_{\max} = 0.7$





10%/30% $f_{\text{obj}} = 369.03$



20%/20% $f_{\text{obj}} = 321.36$



30%/10% $f_{\text{obj}} = 283.12$ Min



$0.3 \le \eta \le 0.7$ $f_{\text{obj}} = 321.36$ Min



$0.3 \le \eta \le 0.6$ $f_{\text{obj}} = 350.08$



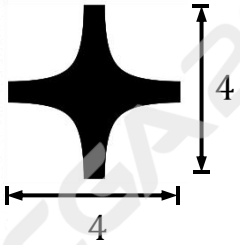
$0.3 \le \eta \le 0.5$ $f_{\text{obj}} = 366.21$

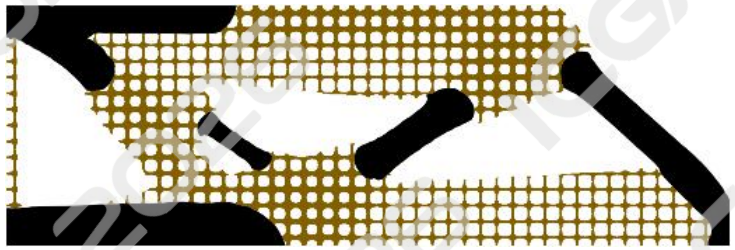
REMARKS: ① customization of phase volumes and lattice sizes ② low mechanical efficiency of lattice vs. solid

Benchmark half-MBB beam design (compliance Min.)

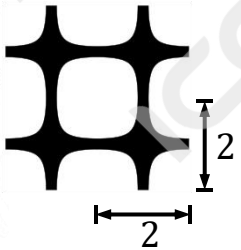


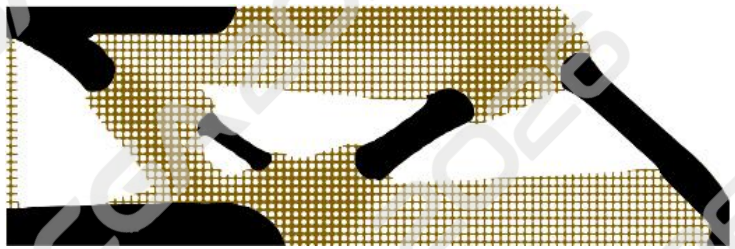
$0.3 \leq \eta \leq 0.6 \quad f_{\text{obj}} = 350.08$





STL model, ready for AM

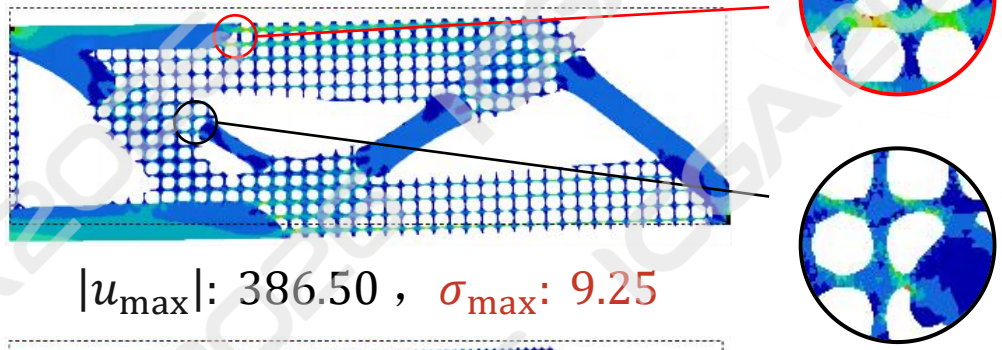




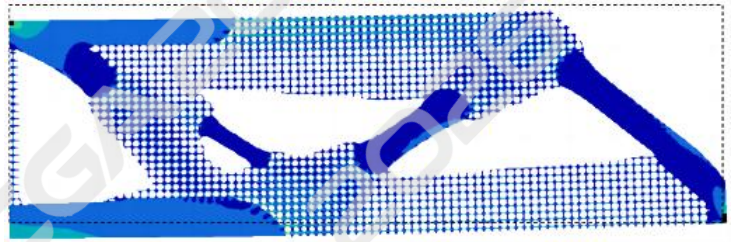
STL model, ready for AM



full-scale FE model



$|u_{\max}|$: 386.50 , $\sigma_{\max}$: 9.25



$|u_{\max}|$: 352.00 , $\sigma_{\max}$: 11.74

REMARKS: ① slight deviation of load-bearing capacity ② latticing scheme dependent mechanical performances

提纲
Outline

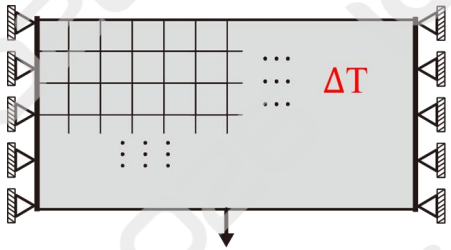
求是创新 明德厚学

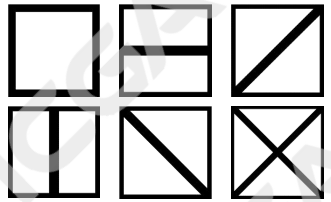
华中科技大学
中国 武汉

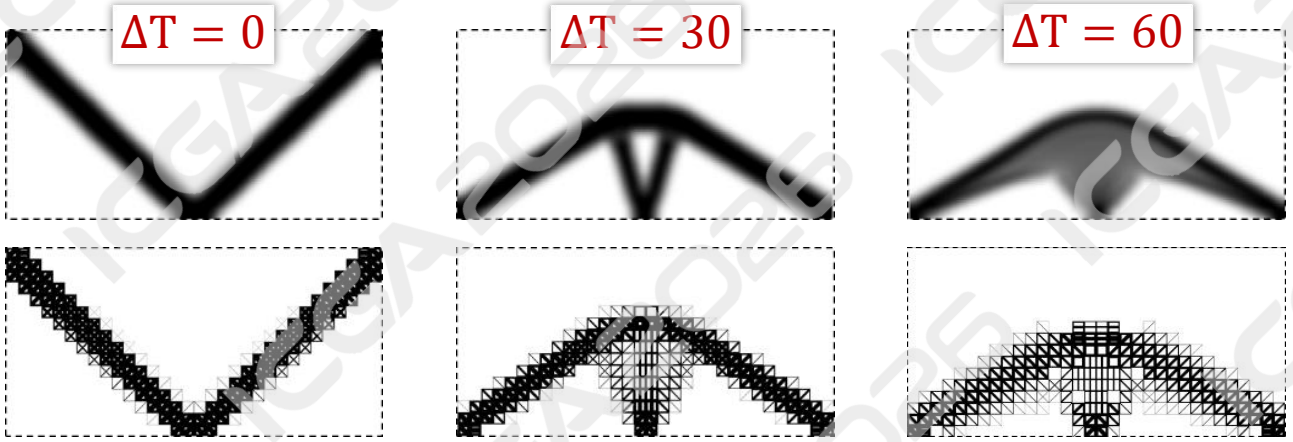
1. Overview & Motivation
2. Multi-material interpolation models
3. Design method for hybrid structures
4. **Applications: thermo-mech coupling & buckling**
5. Conclusion

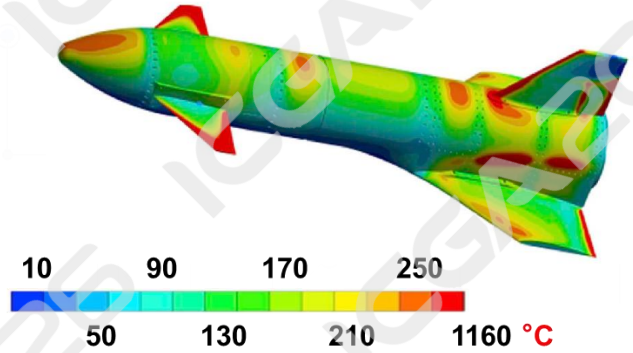
Extension 1: thermal-mechanical designs

- Under high temperatures, gradient/graded lattice structures could match the load-bearing performance of grayscale designs, but with clearer interpretability for manufacturing.
- It is worthwhile to further explore this design method for thermo-mechanical problems, as the solid frame significantly enhances load-bearing capacity compared to pure lattice structures.

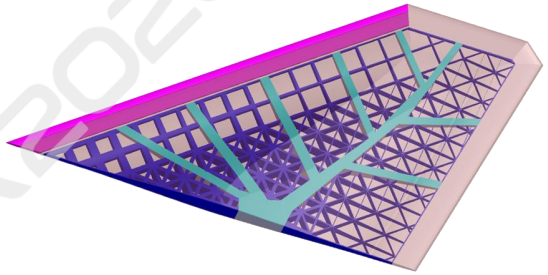








DLR Reusable Launch Vehicle



conceptual rudder design

Bauer et al. Acta Astronautica 2020, Liu et al. SMO 2020, Wang et al. Acta Aero Astro Sinica 2024 (in Chinese)

Hybrid material interpolation model

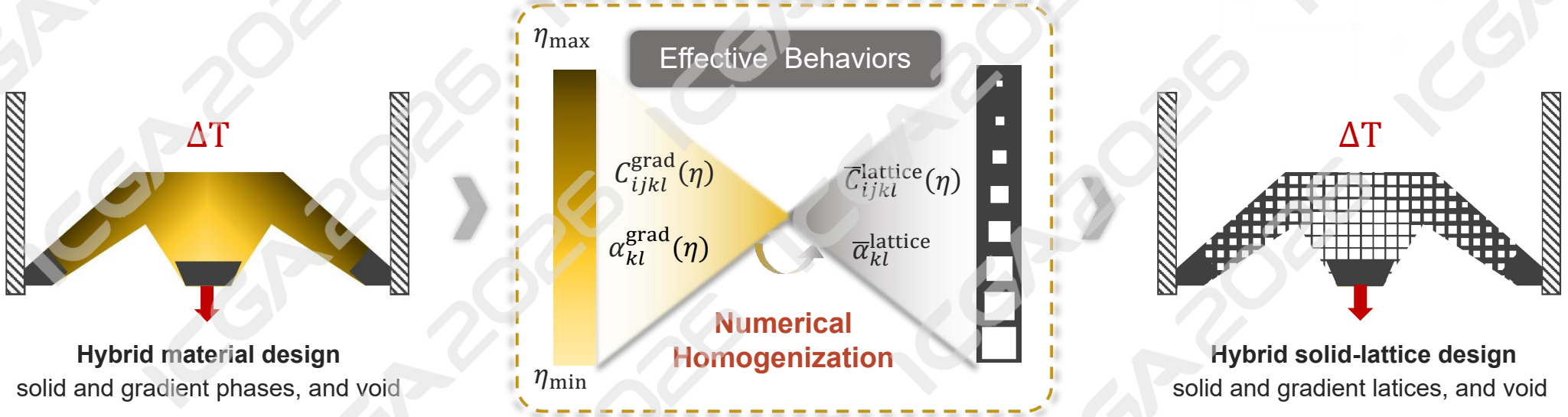
- Extension of the **recursive SIMP** multi-material model

elasticity tensors: $C_{ijkl}(\varphi_1, \varphi_2, \eta) = \varphi_1^p \left(\varphi_2^p C_{ijkl}^{\text{solid}} + (1 - \varphi_2^p) \overline{C}_{ijkl}^{\text{lattice}}(\eta) \right)$

therm exp coeffs: $\alpha_{kl}(\varphi_1, \varphi_2, \eta) = \varphi_1 \left(\varphi_2 \alpha_{kl}^{\text{solid}} + (1 - \varphi_2) \overline{\alpha}_{kl}^{\text{lattice}}(\eta) \right)$

φ_1 : indicating material existence, φ_2 : interpolating the behaviors between solid and lattices

η : gradient control parameter linking to lattice feature geometry size



Li et al. Composite Structures, 2026, 379:119998, **Ian Marshal best paper Award**

Hybrid material interpolation model

- Extension of the **recursive SIMP** multi-material model

thermal expansion coefficients: $\alpha_{kl}(\varphi_1, \varphi_2, \eta) = \varphi_1 \left(\varphi_2 \alpha_{kl}^{\text{solid}} + (1 - \varphi_2) \bar{\alpha}_{kl}^{\text{lattice}}(\eta) \right)$

Levin's Relation (1967)

$$\bar{\alpha} = \alpha_1 + \frac{K_2(\alpha_2 - \alpha_1)}{1 - K_2/K_1} \left(\frac{1}{\bar{K}} - \frac{1}{K_1} \right) \rightarrow \text{given that } K_2 = 0 \quad \bar{\alpha} = \alpha_1$$

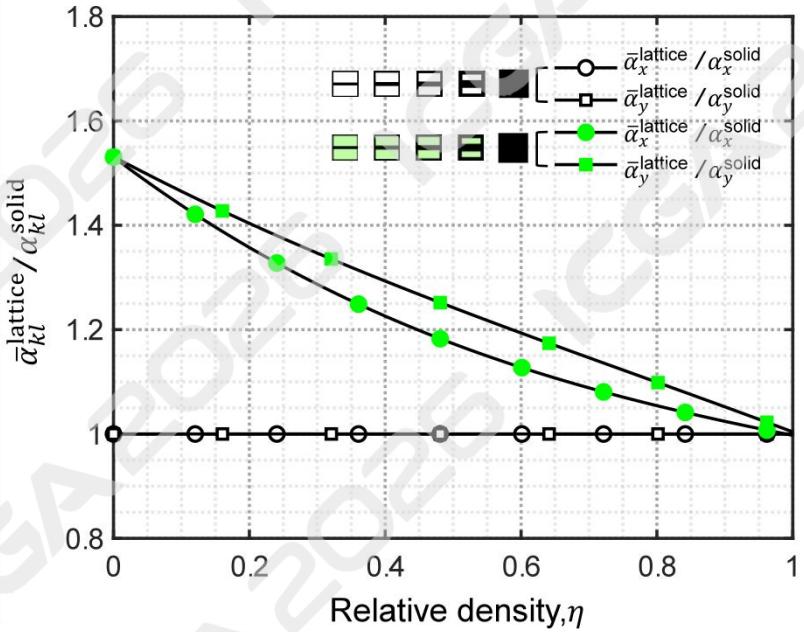
$$\rightarrow \quad \bar{\alpha}_{kl}^{\text{lattice}}(\eta) = \alpha_{kl}^{\text{solid}}$$

Sigmund and Torquato, 1997

$$\rightarrow \quad \alpha_{kl}(\varphi_1, \varphi_2, \eta) = \alpha_{kl}^{\text{solid}}$$

mixture of solid and void

independent of the three variables



black: $E = 199.5\text{GPa}, \alpha = 1.54 \times 10^{-5}\text{°C}^{-1}$
green: $E = 68.9\text{GPa}, \alpha = 2.36 \times 10^{-5}\text{°C}^{-1}$

Levin, VM. Mechanics of Solids, 1967, Sigmund and Torquato, JMPS 1997

Hybrid material optimization model

$$\min : f_{\text{obj}} = \mathbf{U}^{\mathrm{T}}\mathbf{K}\mathbf{U}$$

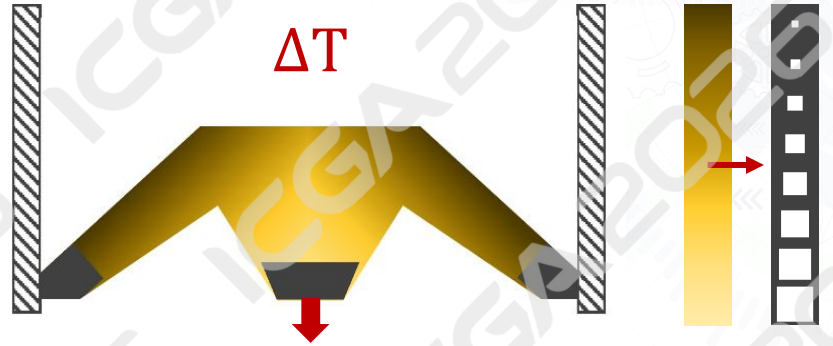
$$\text{subject to} : \mathbf{K}\mathbf{U} = \mathbf{F}_{\text{mech}} + \mathbf{F}_{\text{thermal}}$$

$$: g_{\text{volume}}(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \boldsymbol{\eta}) \leq 0$$

$$: 0 \leq \varphi_{1,e} \leq 1 \, , e = 1, \ldots, N$$

$$: 0 \leq \varphi_{2,e} \leq 1 \, , e = 1, \ldots, N$$

$$: \eta_{\min} \leq \eta_e \leq \eta_{\max} \, , e = 1, \ldots, N$$



$$g_{\text{volume}} = \frac{V_{\text{solid}} + V_{\text{lattice}}}{(\vartheta_{\text{solid}} + \vartheta_{\text{lattice}})\, V_{\text{domain}}} - 1 \leq 0$$

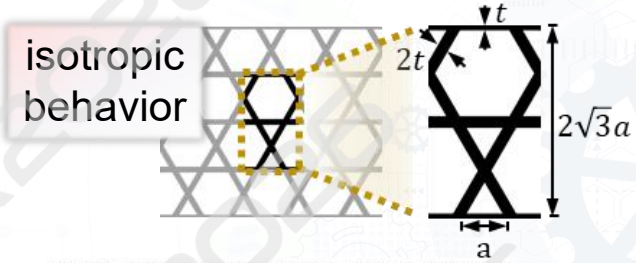
ϑ_{solid} : volume fraction of the primary solid material phase over the design domain

$\vartheta_{\text{lattice}}$: volume fraction of the lattices over the design domain

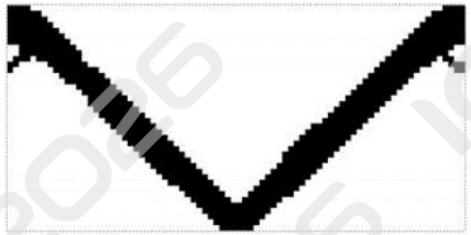
❖ One unified global volume constraint: adaptive volume distribution of solid and lattice materials

separate volume constraints: $g_{\text{solid}} = \frac{V_{\text{solid}}}{\vartheta_{\text{solid}} V_{\text{domain}}} - 1 \leq 0, \quad g_{\text{lattice}} = \frac{V_{\text{lattice}}}{\vartheta_{\text{lattice}} V_{\text{domain}}} - 1 \leq 0$

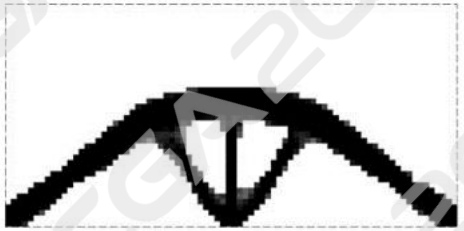
Benchmark two-clamped beam problem



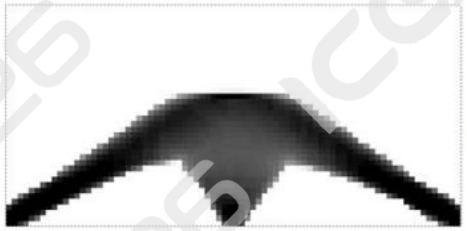
- SIMP model with isotropic behavior assumption



$f_{\text{obj}} = 1618.6$



$f_{\text{obj}} = 2940.5$



$f_{\text{obj}} = 4728.7$



$f_{\text{obj}} = 6849.0$

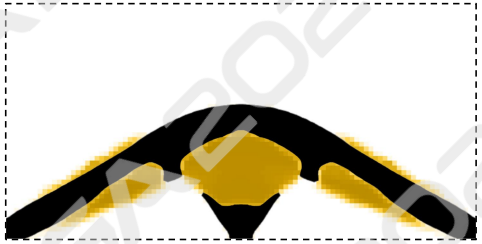
- Hybrid solid-lattice design 【global volume: 20%, $0.2 \leq \eta \leq 0.6$】



$\vartheta_{\text{solid}} = 19.7\%$ $\vartheta_{\text{lattice}} = 0.3\%$
$f_{\text{obj}} = 1685.6$



$\vartheta_{\text{solid}} = 19.8\%$ $\vartheta_{\text{lattice}} = 0.2\%$
$f_{\text{obj}} = 2774.3$



$\vartheta_{\text{solid}} = 14.3\%$ $\vartheta_{\text{lattice}} = 5.6\%$
$f_{\text{obj}} = 4622.2$



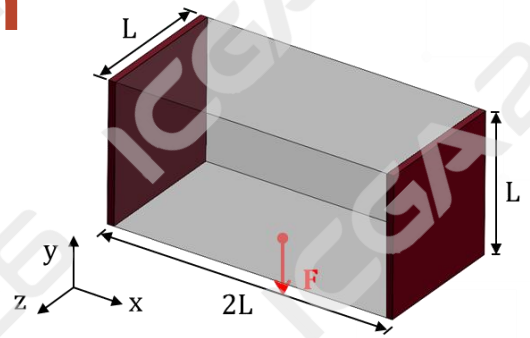
$\vartheta_{\text{solid}} = 5.5\%$ $\vartheta_{\text{lattice}} = 14.2\%$
$f_{\text{obj}} = 6969.3$

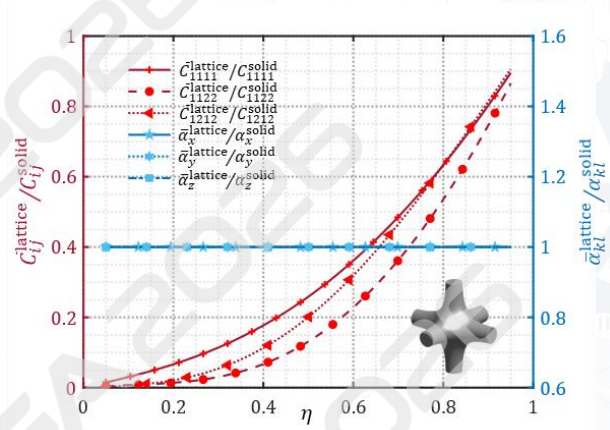
$\Delta T = 0℃$ $\Delta T = 5℃$ $\Delta T = 10℃$ $\Delta T = 15℃$

REMARKS: resembling the SIMP designs, with comparable performance yet clearer physical interpretation

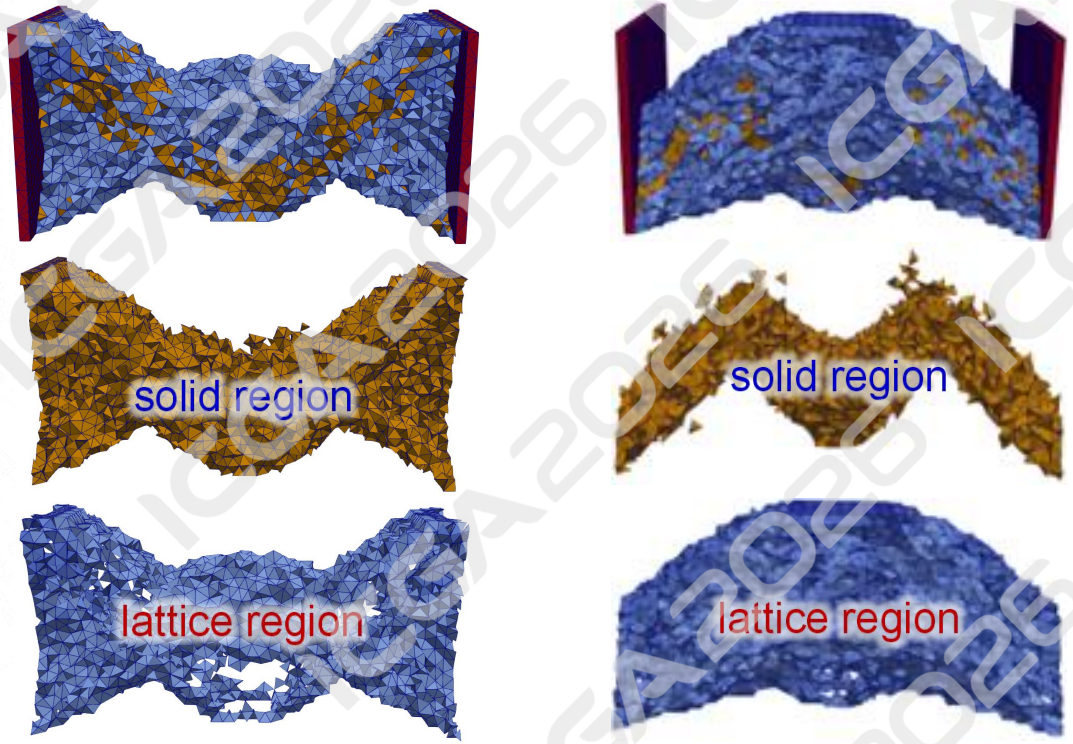
3D design problem extension

volume fraction:	$\vartheta_{\text{solid}} + \vartheta_{\text{lattice}} = 30\%$
lattice density ranges:	$\eta_{\min} = 0.2,\ \eta_{\max} = 0.5$





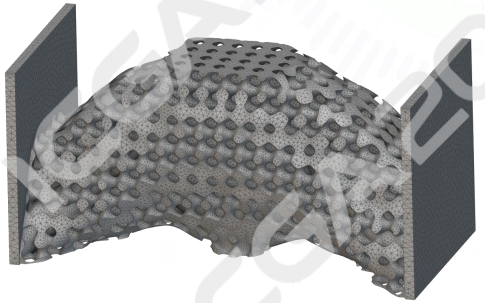
$\Delta T = 50℃$ $\Delta T = 150℃$



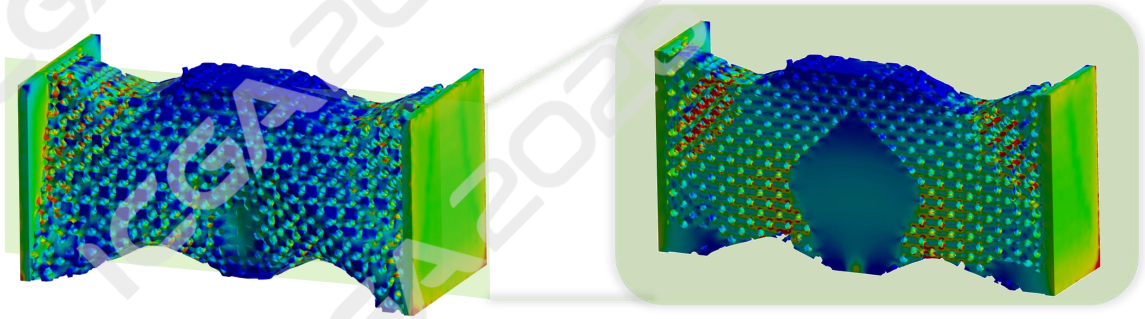
$\vartheta_{\text{solid}} = 25.23\%$ $\vartheta_{\text{lattice}} = 4.77\%$ $\vartheta_{\text{solid}} = 13.09\%$ $\vartheta_{\text{lattice}} = 16.90\%$



coarsely latticed FE model



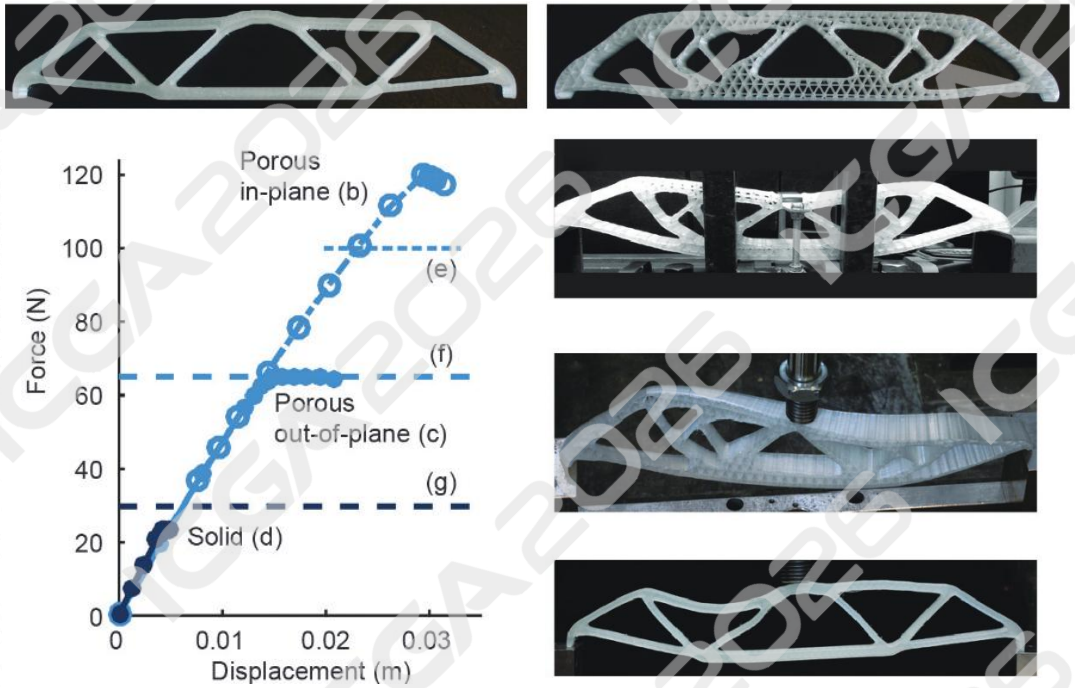
finely latticed FE model



$|u_{\max}| = 0.44\ \text{mm},\ \sigma_{\max} = 8.07 \times 10^3 \text{MPa}$

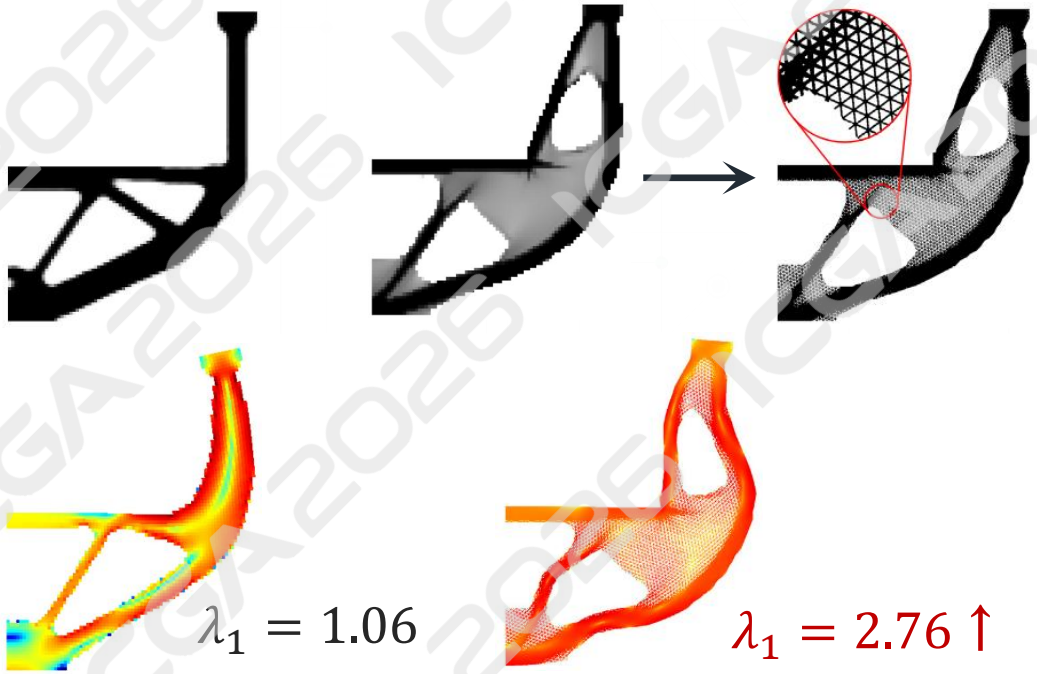
Extension 2: buckling-resistant designs

- Solid frame structures provide global stiffness, while gradient/graded lattice provides distributed support to slender regions, thereby delaying instability.
- Literature reported that higher critical buckling loads for hybrid solid-lattice structural form.



Experimental Validation of Infill-Enhanced Buckling Resistance

C.F. Christensen, F. Wang, O. Sigmund. CMAME 2023



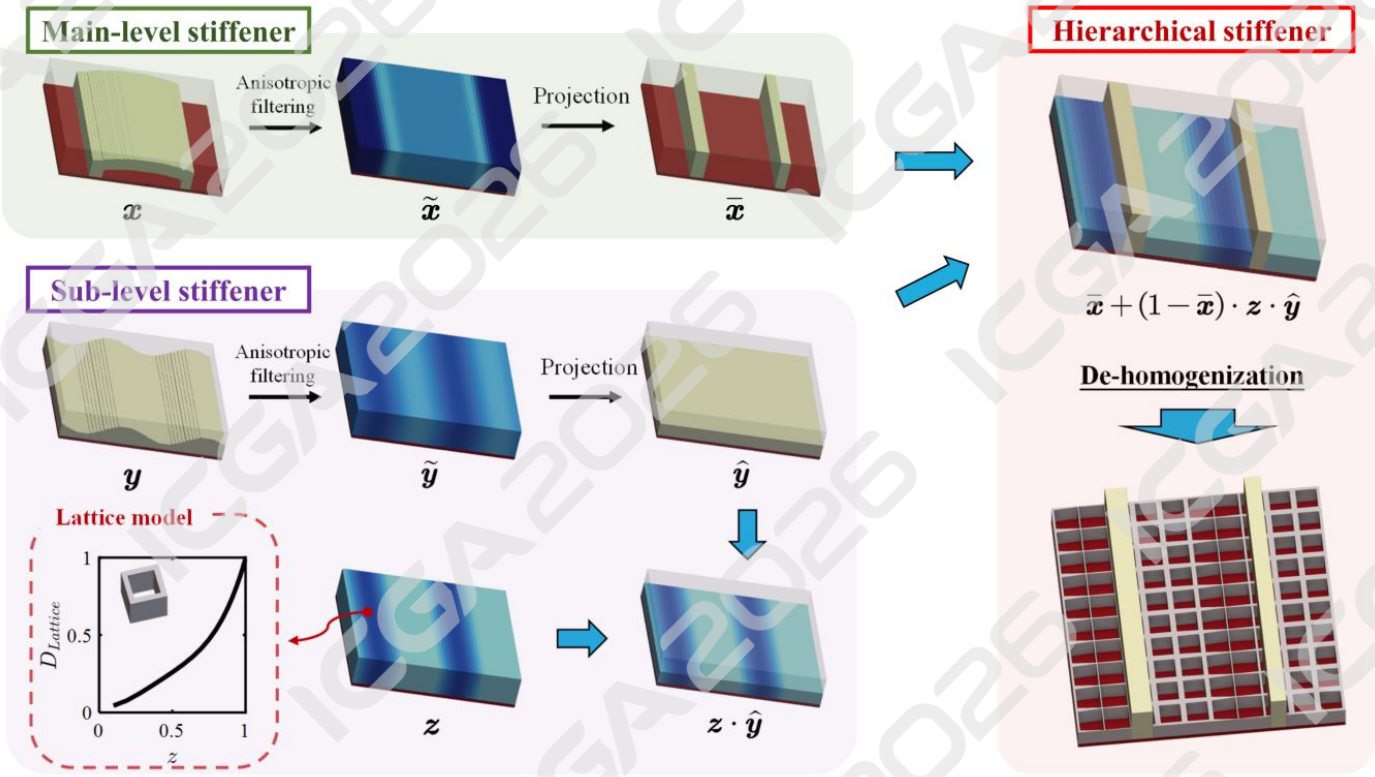
Buckling Optimization of Solid–Lattice Hybrid Structures

A. Clausen, N. Aage, O. Sigmund. Engineering 2016

Hybrid material model for hierarchical stiffeners

- Extension of the **hybrid material model** with additional height control parameter

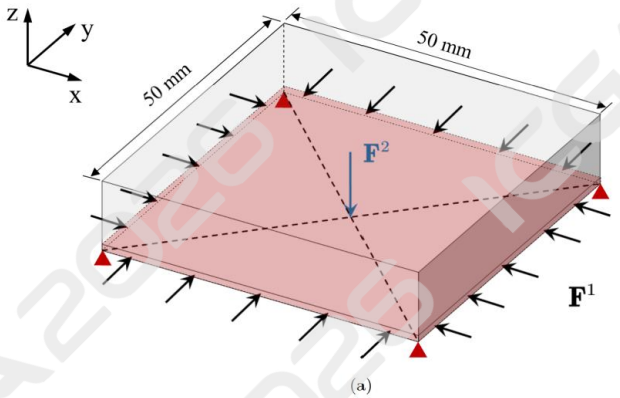
elasticity tensors: $C_{ijkl}(\bar{x},\hat{y},\eta) = \bar{x}^p C_{ijkl}^{\text{solid}} + (1-\bar{x}^p)\hat{y}^p \overline{C}_{ijkl}^{\text{lattice}}(\eta)$

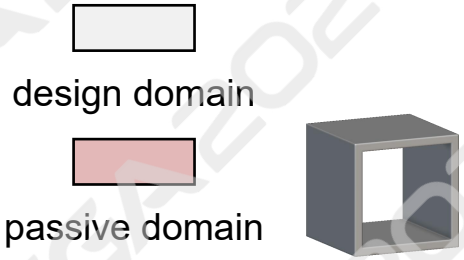


The density field $\hat{y}$ facilitates the height-control constraint, regulating the height of the sub-level stiffeners.

$\bar{x} = 1$	Solid phase
$\bar{x} = 0,\ \hat{y} = 1$	Lattice phase
$\bar{x} = 0,\ \hat{y} = 0$	Void phase

Planar stiffened structure design



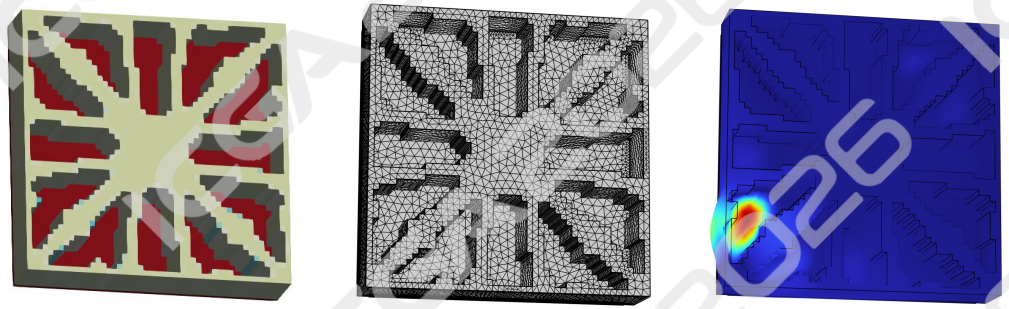


$\mathbf{F}^1$: buckling load factor

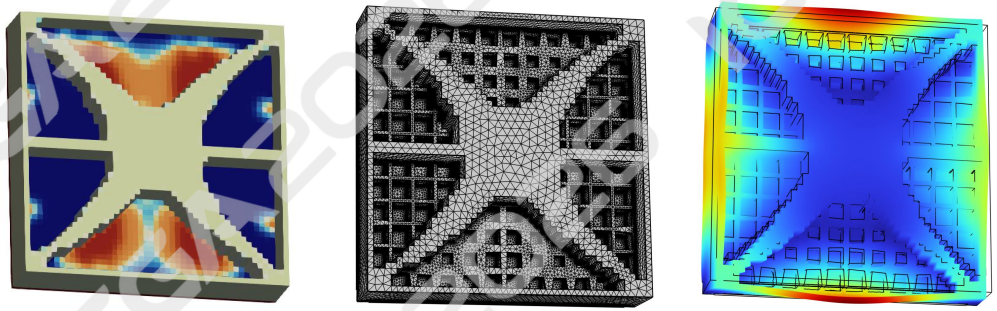
$\mathbf{F}^2$: structural compliance

$$\max:\ \min(\lambda)$$
$$s.t.\ :\ V(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\eta}) \leqslant 0$$
$$c = (\mathbf{F}^2)^{\top}\mathbf{U}^2 \leqslant \bar{c}$$
$$\mathbf{K}\mathbf{U}^1 = \mathbf{F}^1$$
$$\mathbf{K}\mathbf{U}^2 = \mathbf{F}^2$$
$$[\mathbf{K} + \lambda_n \mathbf{K}_\sigma^1]\boldsymbol{\varphi}_n = 0$$
$$0 \leqslant x_e, y_e \leqslant 1,\ e = 1,...,N_e$$
$$\eta_{\min} \leqslant \eta_e \leqslant \eta_{\max},\ e = 1,..,N_e$$

Young's modulus:	210 GPa
Poisson's ratio:	0.3
F1:	400 N
F2:	1000 N



pure solid design: $c = 11.0,\ \lambda_1 = 6288.5$



hybrid solid-lattice design: $c = 11.0,\ \lambda_1 = 8421.7 \uparrow$

$\boldsymbol{\eta} \in [0.3,\ 0.8]$

提纲
Outline

求明德厚学
是创新

华中科技大学
中国 武汉

1. Overview & Motivation
2. Multi-material interpolation models
3. Design method for hybrid structures
4. Applications: thermo-mech coupling & buckling
5. **Conclusion**

Conclusion

- This work proposes a multi-material topology optimization approach for the design of lightweight hybrid solid-lattice structures, with preliminary explorations on thermal-mechanical coupling and buckling-resistant design problems, featuring the following key insights:
  - The formulated hybrid material model enhances design flexibility by enabling the customization of material compositions and gradient variable ranges.
  - Expanding the design space does not inherently guarantee superior performance; instead, **it increases non-linearity, leading to convergence and local optima difficulties.**
- The benefits of hybrid structural forms integrating lattices with solid frame structures warrant further exploration in broader physical scenarios, such as dynamic, acoustic, and fluid-structure interaction (FSI) problems with functional lattices.

1) A multi-material topology optimization approach to hybrid material structures with gradient lattices, **Comput Methods Appl Mech Eng**, **2024**, 425:116969
2) Hybrid material topology optimization of solid-lattice structures for natural frequency maximization, **Advances in Engineering Software**, **2025**, 208:103961
3) Thermal-mechanical design of hybrid solid-lattice structures using multi-material topology optimization, **Composite Structures**, **2026**, 379:119998
4) Hierarchical stiffened design of thin-walled structures using a density-based topology optimization method, **Thin-walled Structures**, submitted

END

Thanks for Your Attention

XIA Liang / xialiang@hust.edu.cn

School of Mechanical Science and Engineering
Huazhong University of Science and Technology