Structural Topology Optimization for Acoustic Black Hole (ABH)

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HYU HANYANG UNIVERSITY SEOUL, KOREA

MPSD 멀티피직스 연구실
MultiPhysics System Design laboratory

Structural Optimization

PHM with AI

Expandable Soft Robot

Particle contact and optimization 최적화

World-Class Achievements in Hyperadaptive Multiphysics System Analysis and Optimal Design



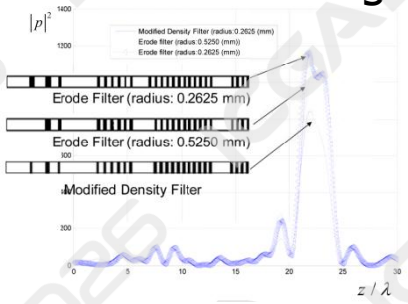


Inbound and outbound

- ✓ World-Class Achievements in Analysis and Design
- ✓ General Member for IUTAM (Solid)
- ✓ Member of the Executive Committee of International Society for Multidisciplinary Optimization (ISSMO), serving as a representative for Asia
- ✓ Journal of Mechanical Science and Technology, Editor in Chief
- ✓ Vice Dean of the Graduate School of Engineering, Hanyang University; Member of the Faculty Council
- ✓ As of 2026: 174 SCI-indexed journal publications; H-index: 41
  Research funding: 76 projects, totaling KRW 6.0 billion

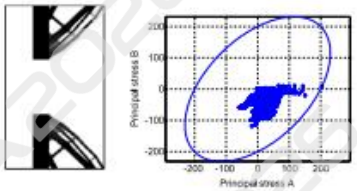
Multiphysics (FSI, Acoustic-Structure, Turbulent, PZT)

PZT & Acoustic Design



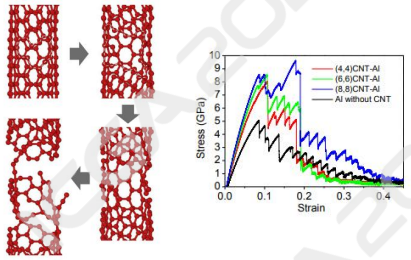
[Yoon et al., CMAME, 2016]

FSI- stress Design



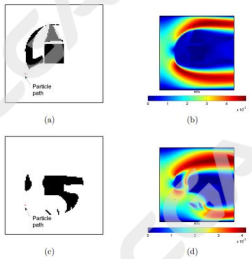
[Yoon, CMA, 2017]

MD Simulation



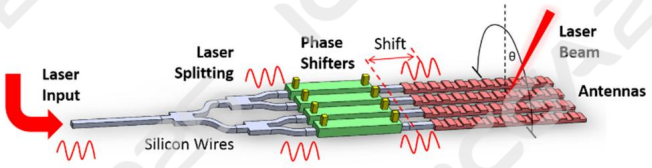
[Choi et al., Composites part B, 2016]

Particle and flow



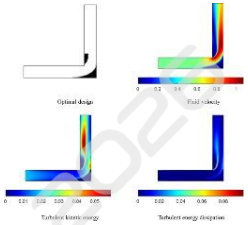
[Yoon, CMAME, 2020]

Opto-electronics



[Yoon et al.,IJHMT, 2020]

Turbulent flow

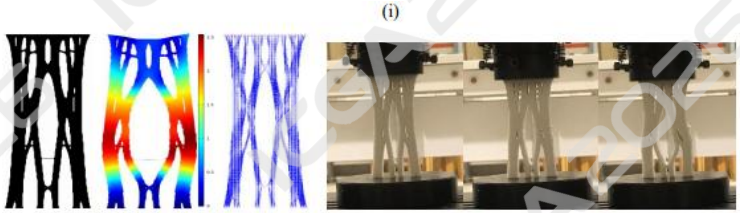


[Yoon, CMAME, 2020]

Contribution: Modifications of governing equations for design o ptimization based on continuum mechanics theory

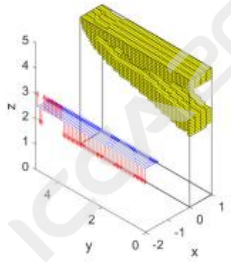
Solid Mechanics: Design for manufacturing

3D printing Buckling



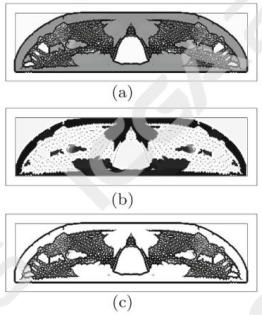
[Bing, et al. Additive manufacturing, 2021]

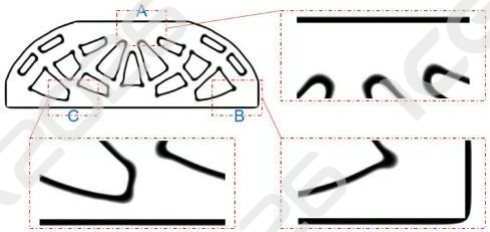
Overhang constraint



[Yoon, Computers and Structures, 2021]

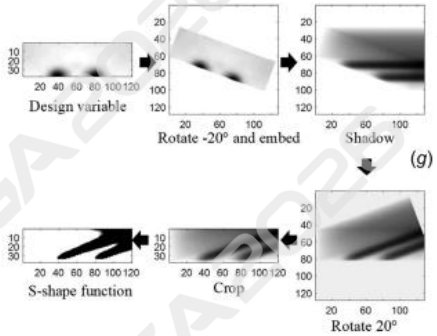
Coating Design





[Yoon and Bing , SMO, 2019]
[Bing, et al. ,Engineering optimization, 2021]

Manufacturing Design



[Yoon, et al. ,J. Mechanical Design, 2022]

Design for Manufacturing and Additive Manufacturing

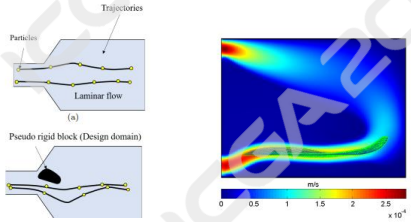
Solid Mechanics: Robotics and Micro System Application

Ultrasound wave focusing



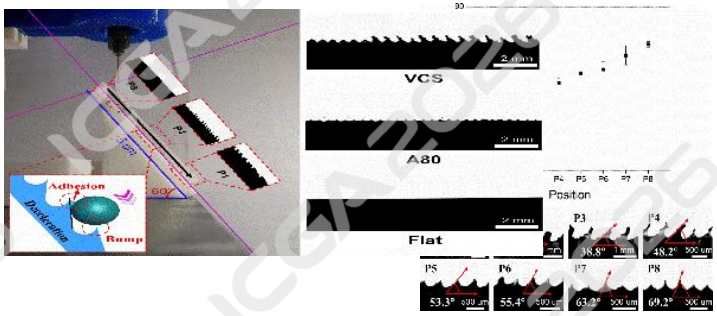
[Hur, et al. Scientific Report, 2022]

Microparticle application



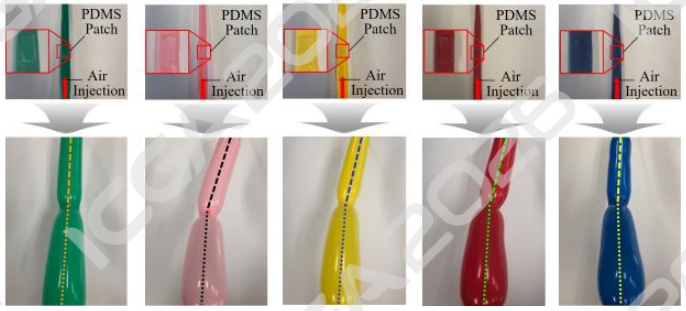
[Yoon, CMAME 2020, Yoon, SMO, 2022]

Wettability, hydrophobic,
3d printer



[Sung, et al. JPGT, 2022]

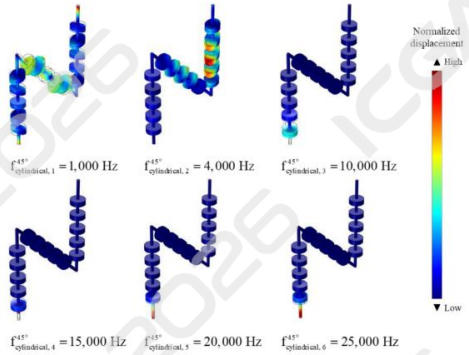
Soft robot & PDMS



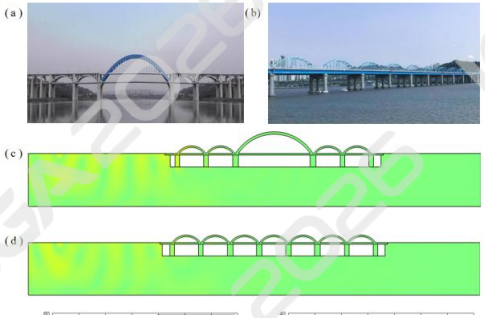
[Lee, et al. JMBBM, 2021]

Design for robotics and micro application

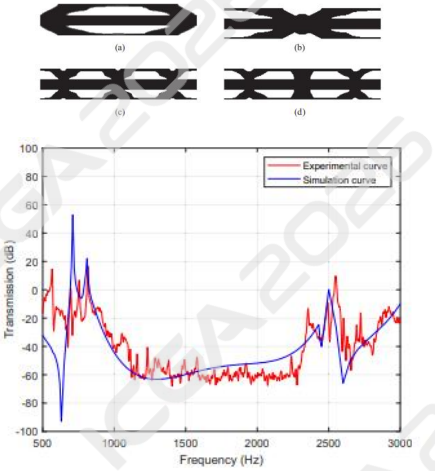
Solid Mechanics: Elastic and acoustic metamaterials



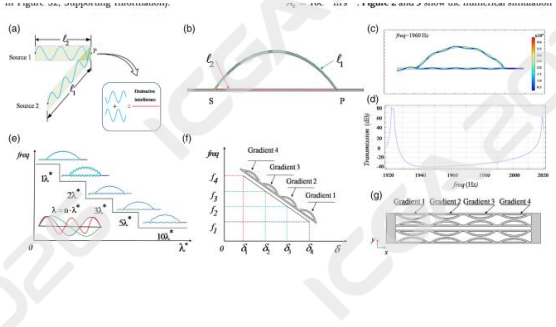
[Yoon, et al. ,Applied science, 2021]



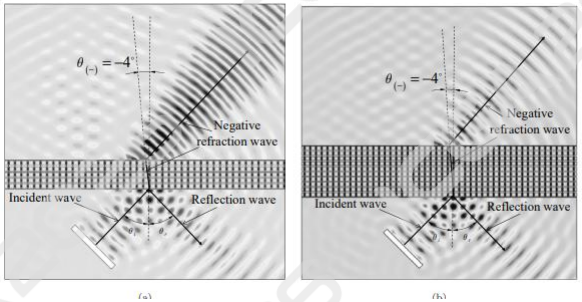
[Liu, et al. ,Extreme Mechanics letter, 2020]



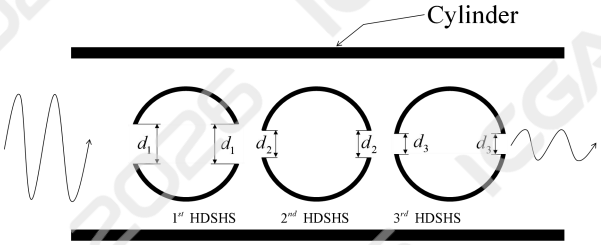
[W. Liu et al. Computers and Structures,2020]



[Liu, et al. ,Advanced Engineering Materials, 2020]



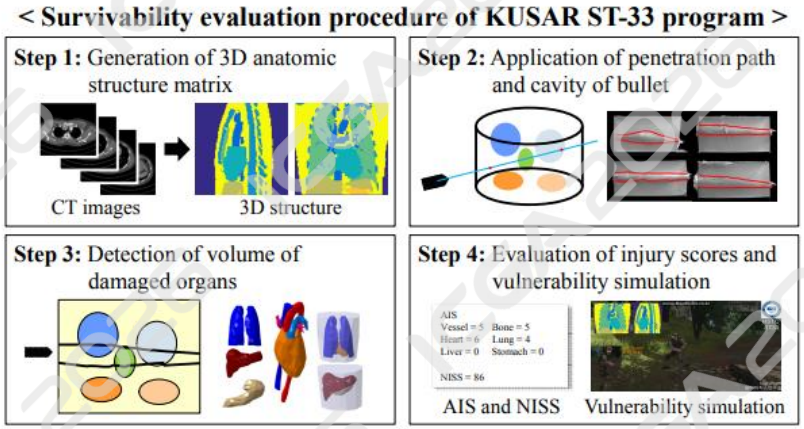
[Choi and yoon, J. Theoretical and computational acoustics, 2018]



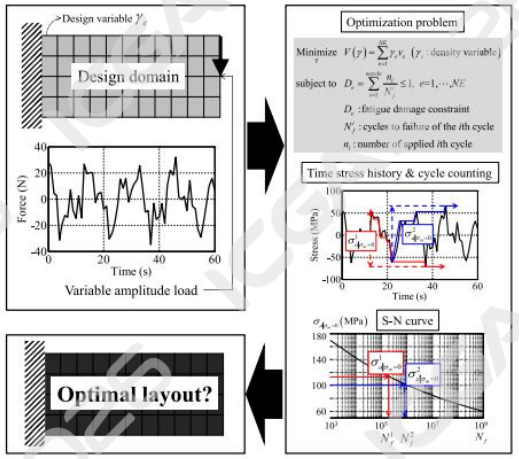
[Choi and Yoon, J. Vibration and control, 2018]

Design optimization approach for acoustic, vibration, wave propagation and metamaterials

Solid Mechanics: Survivability and defense engineering



[Yoon et al., JMST, 2018]



[Jeong et al., AMM. 2018]

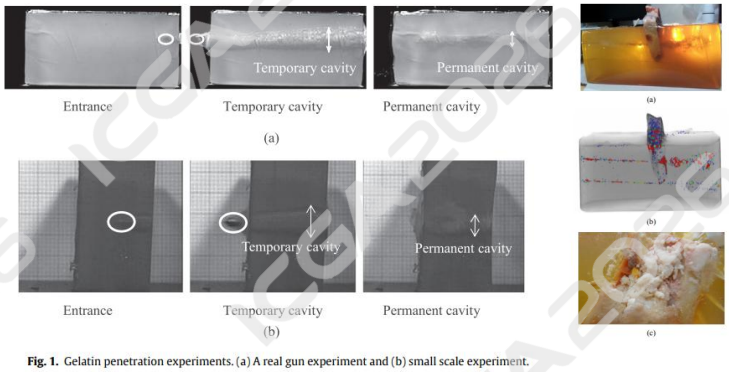
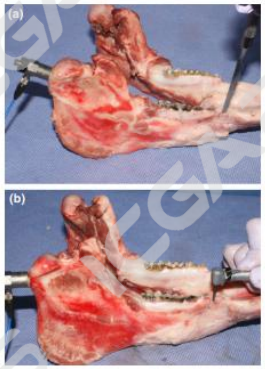


Fig. 1. Gelatin penetration experiments. (a) A real gun experiment and (b) small scale experiment.

[Park et al., J. Impact Engineering.2017]



[Jung et al., Clin Oral Impants Res, 2017]

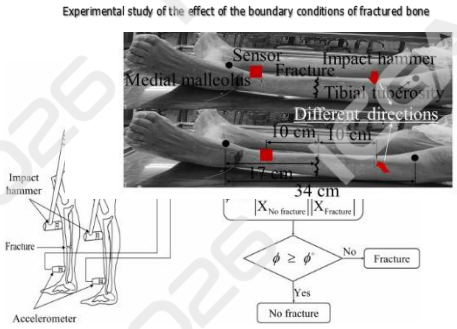


저살상 총기 개발

Experimental study for survivability and defense system

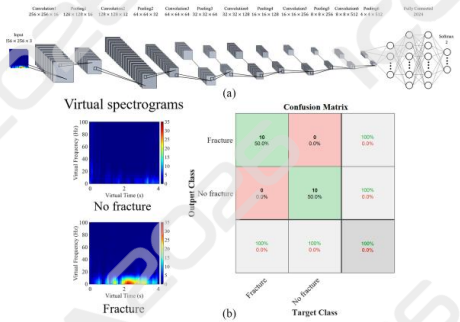
AI-assisted Mechanical Design

신체 이상 반응 측정



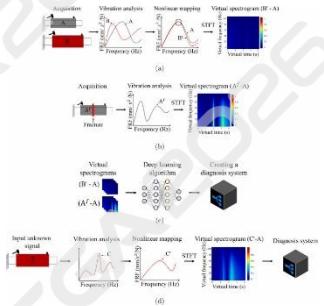
[Sim and Yoon, JMBBM, 2021]]

이상 반응 측정을 위한 Nonlinear mapping



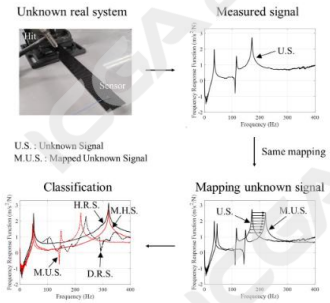
[Yoon et al., JEM, 2021]

Data augmentation (Virtual Spectrogram)



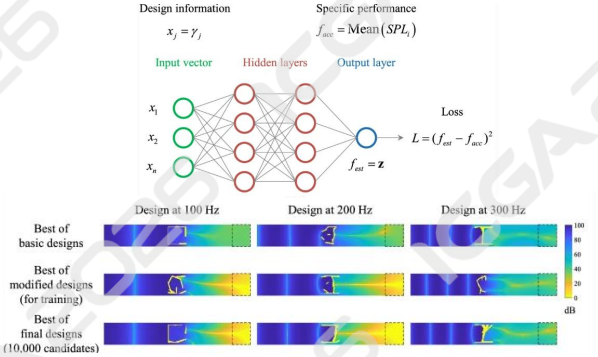
[Kim and Yoon, MSSP, 2022]

Composite Delamination



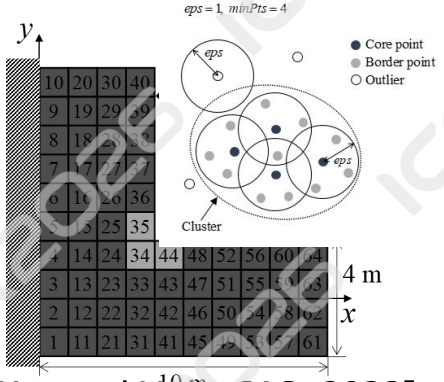
[Kim et.al., under review, 2023]

Acoustic AI 설계



[Kim and Yoon, SMO, 2022]

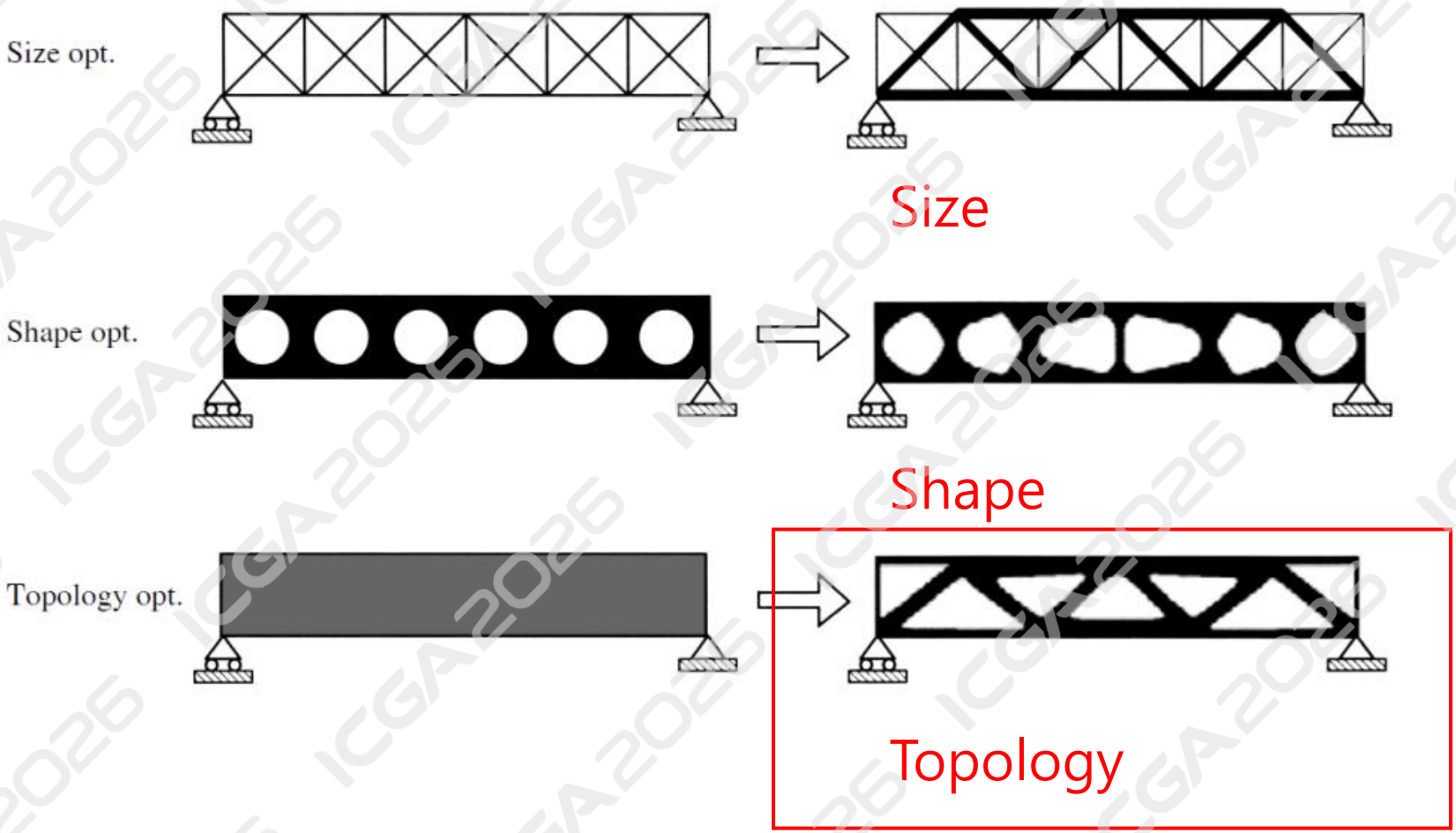
최적설계 기반 이상진단



[Kyo and Yoon,CAS, 2023]

Issue: AI based PHM

Size, Shape, Topology optimization



$$\min_{x} \mathbf{F}^T \mathbf{U}$$

Subject to $\text{Mass} \le M^0$

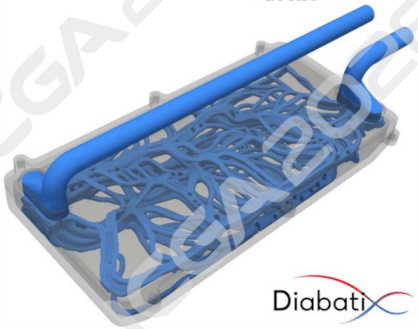
$$\mathbf{K}(\mathbf{X})\mathbf{U} = \mathbf{F}$$

$$0 \le x_i \le 1$$

Void

Solid













Multiphysics problem: Simultaneous interaction of multiple physical phenomena

Noise and vibration

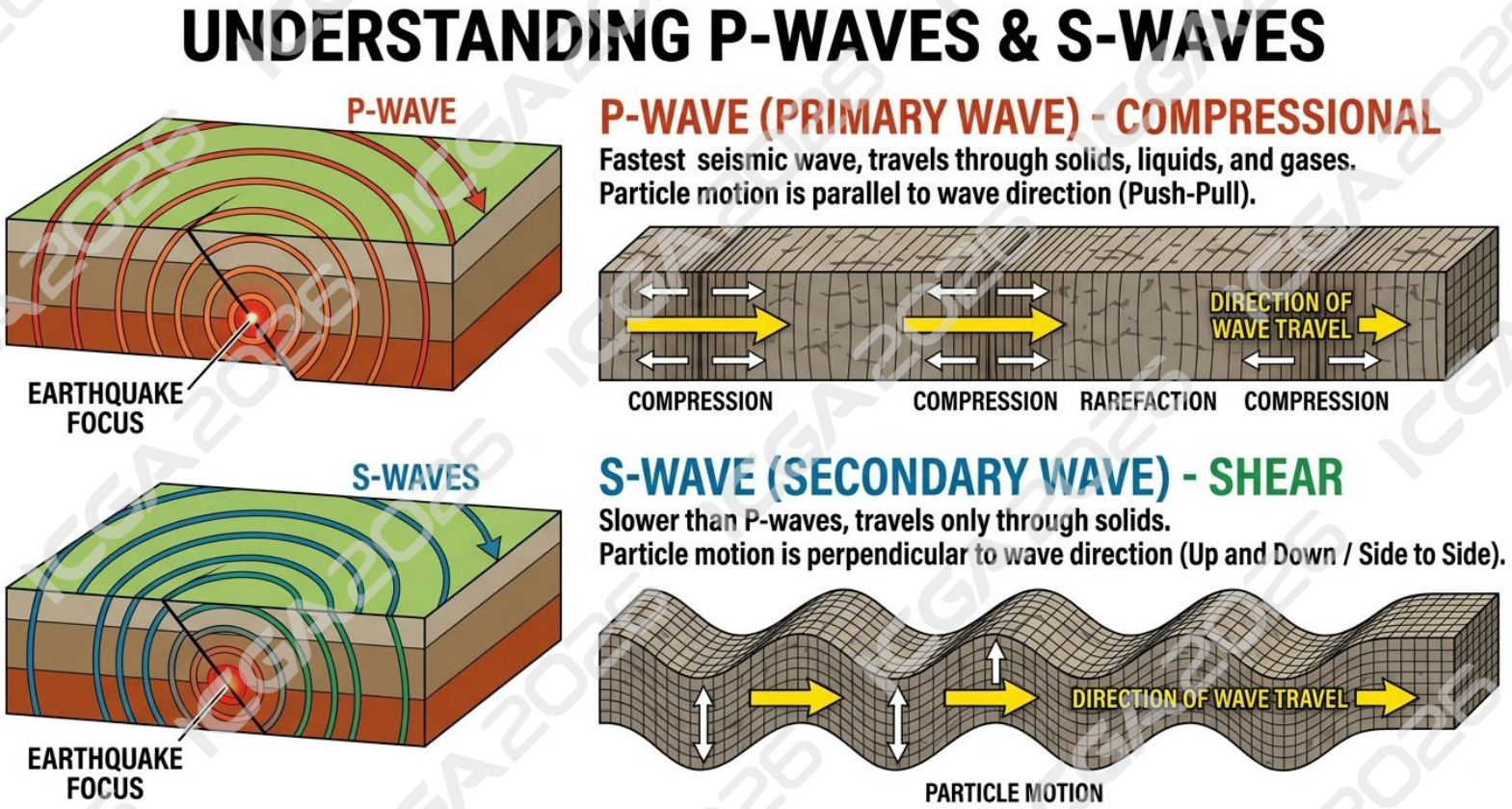




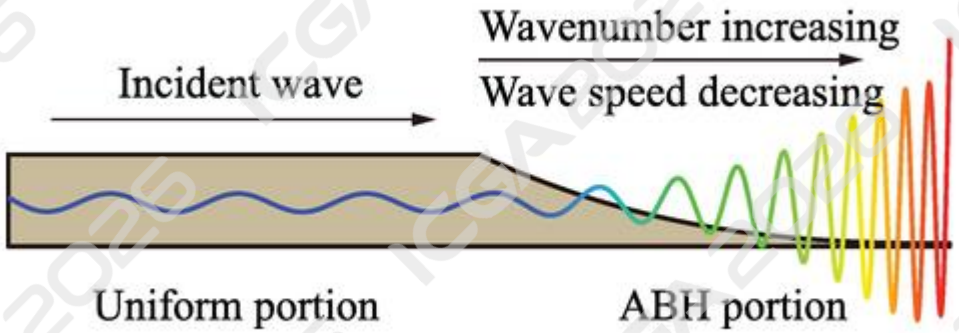

Mode conversion design for ACOUSTIC BLACK HOLE (ABH)

a passive wave-control mechanism in which flexural waves are gradually slowed and spatially localized by a smooth reduction of structural thickness

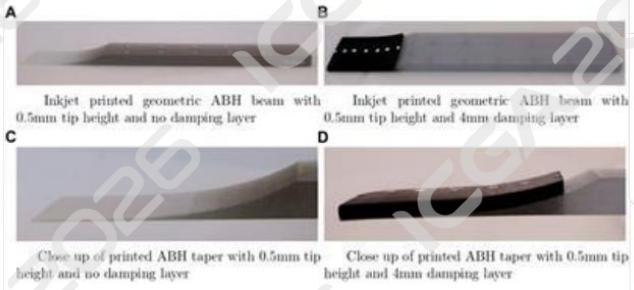
Mechanical wave (P- wave and S-wave)

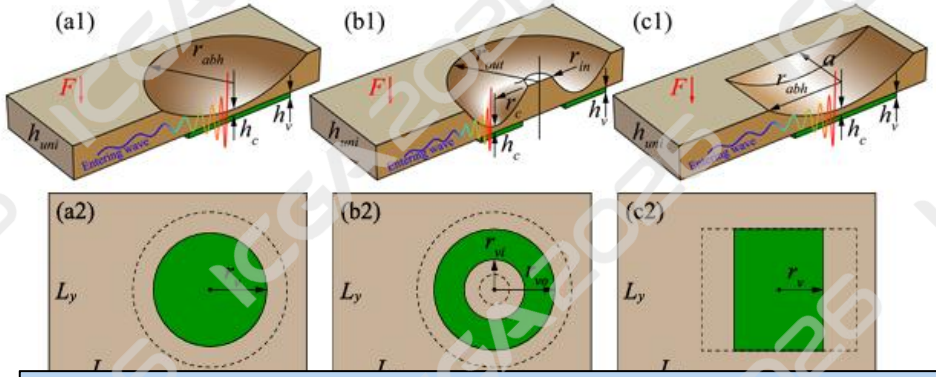


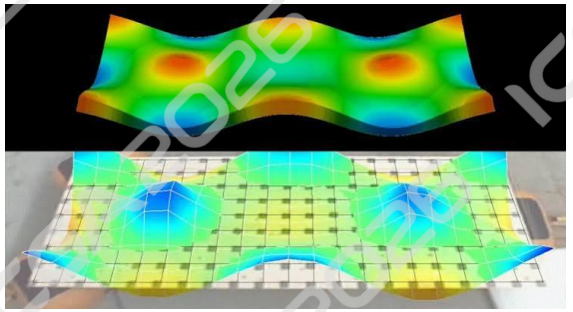
Mode conversion for acoustic black hole (ABH)





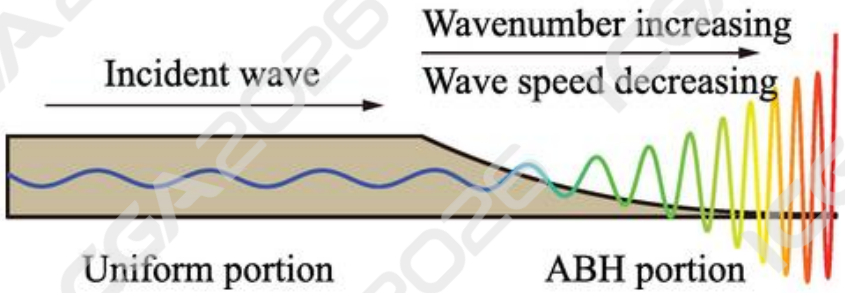






It is a splendid idea to trap vibration.
But does it really work?

Mode conversion for acoustic black hole (ABH)



$$k_F(x) = \left[\frac{\rho A_b(x)\omega^2}{\mathrm{B}(x)}\right]^{1/4} = \left[\frac{12\rho\omega^2}{Eh^2(x)}\right]^{1/4}$$

$$c_F(x) = \frac{\omega}{k_F(x)} = \left[\frac{Eh^2(x)}{12\rho}\right]^{1/4} \omega^{1/2}$$

It only works for transverse wave.
What happen to longitudinal wave?

Mode conversion for acoustic black hole (ABH)

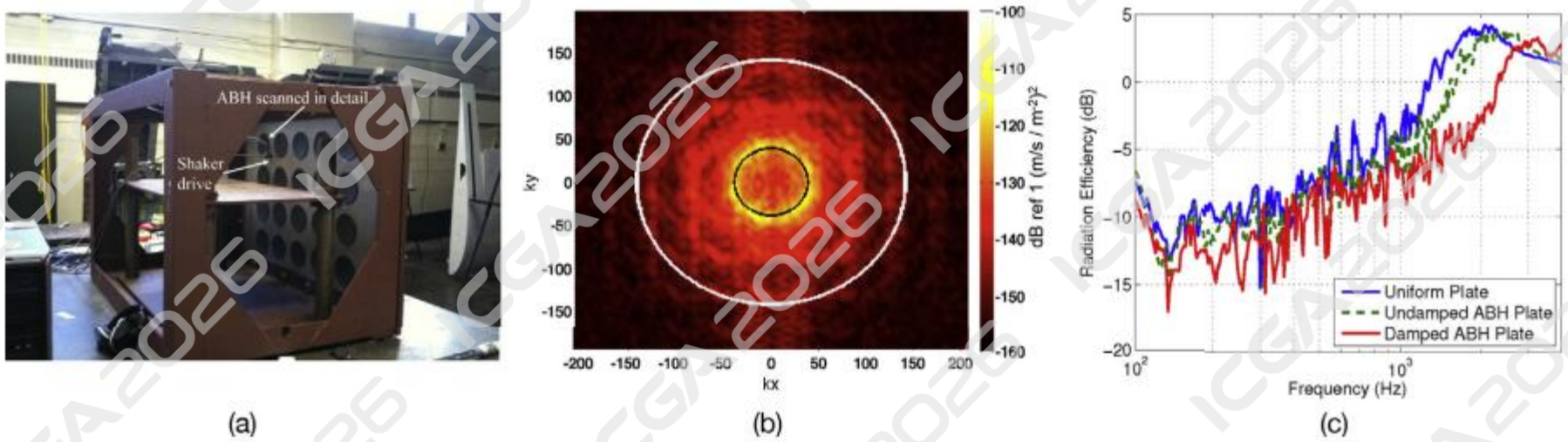
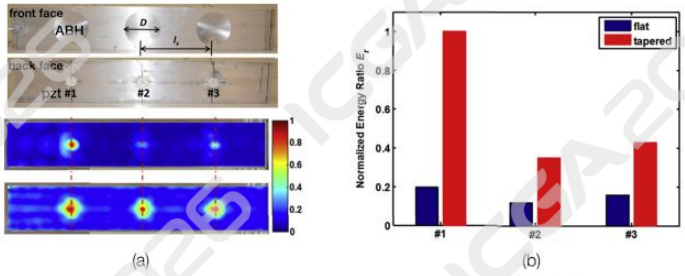


Fig. 11. (a) aluminum plate with an array of embedded ABHs; (b) wavenumber spectrum of an undamped ABH plate at 2500 Hz one third octave band with infinite plate wavenumbers for uniform plate thickness (inner circle) and minimum ABH thickness (outer circle). The vibration energy is spread out over different wavenumbers due to the ABH effect and it is effectively bounded by the wavenumber of the minimum ABH thickness; (c) radiation efficiency of uniform, undamped, and damped ABH plates. Embedding ABH tapers increases the critical frequency of the plate. At 2 kHz the radiation efficiency of the damped ABH plate is 6 dB less than the uniform plate (from Ref. [56]).



(a) top: front and back view of a quasi 1D ABH structure using shunted PZT patches to harvest mechanical energy; bottom: resulting displacement fields at f = 10 kHz
20 kHz acquired via a laser Doppler vibrometer; (b) normalized energy ratio showing the performance of the piezo transducers in terms of energy extraction on a
re with and without ABH tapers (from Ref. [107]).

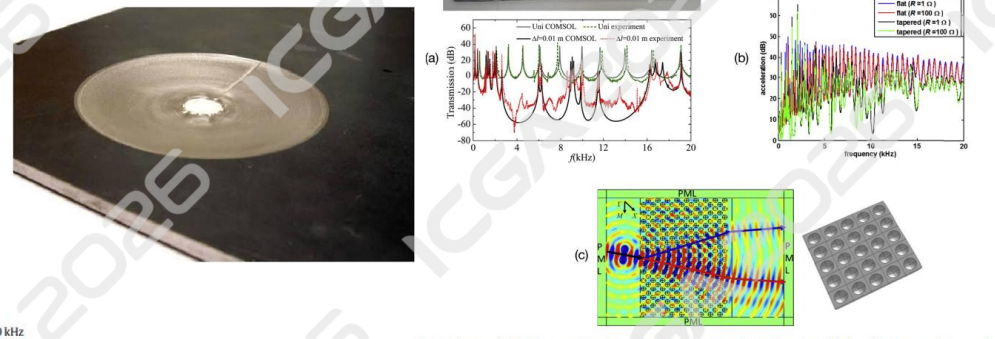
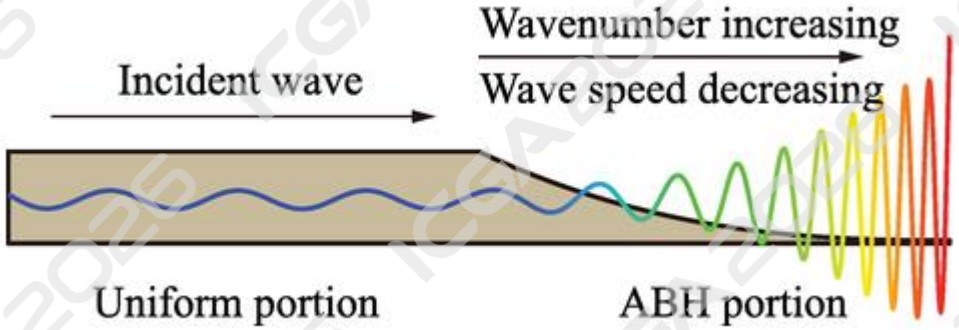


Fig. 16. Various types of multi-ABH structures. (a) top: beam with a grid of enclosed compound ABHs with or without reinforcing stud; bottom: comparison between the transmission obtained via numerical and experimental approaches (from Refs. [95]). (b) top: side view of a plate with a grid of embedded ABHs where PZT patches are glued in correspondence of the tapers and shunted on external resistors; bottom: comparison of acceleration spectra (from Refs. [97]); (c) top: schematic of a phononic ABH plate; bottom: simulated displacement field showing a bi-refractive behavior (from Refs. [102]).

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Mode conversion for acoustic black hole (ABH)





Issues

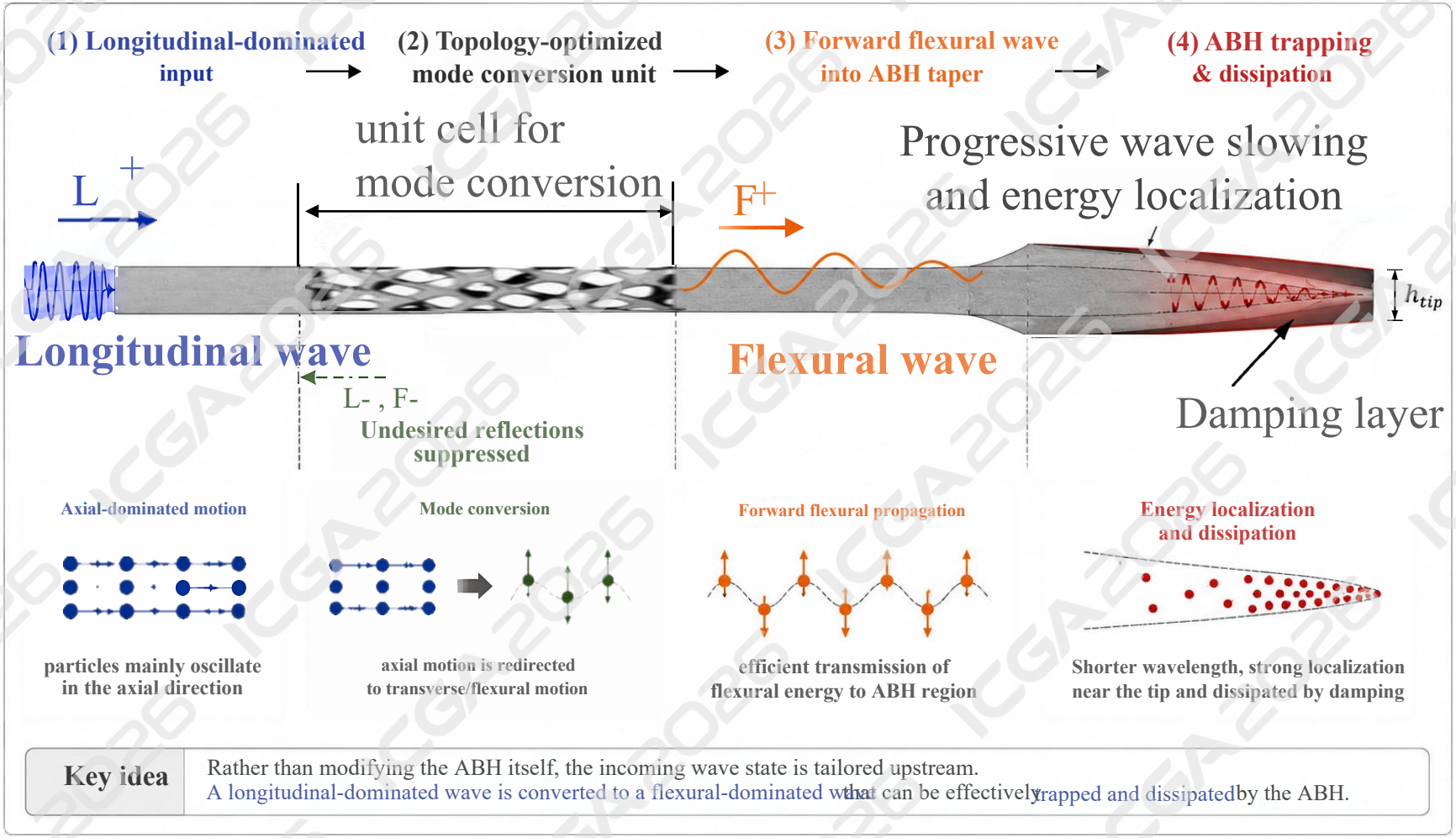
- In practice, a finite residual thickness is unavoidable because of manufacturing and structural constraints, resulting in residual reflection at the tapered tip.

-Moreover, because the taper mainly localizes vibrational energy rather than dissipating it, a damping layer is usually applied near the thin region

- It only works for flexural wave. How about the longitudinal wave?

Mode conversion for acoustic black hole (ABH)

ABH only works for transverse wave.
What happen to longitudinal wave?



Mode conversion for acoustic black hole (ABH)

Helmholtz decomposition:

$$\mathbf{U} = \nabla\phi + \nabla \times \boldsymbol{\psi}$$

Longitudinal wave Transverse wave

Helmholtz's theorem is known as the fundamental theorem of vector calculus, which states that any sufficiently smooth, rapidly decaying vector field in three dimensions can be resolved into the sum of an irrotational (curl-free) vector field and a solenoidal (divergence-free) vector field

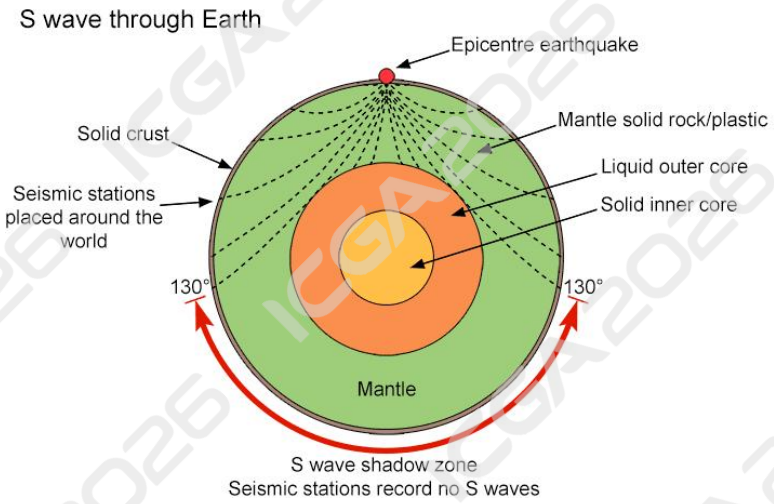
Mode conversion for acoustic black hole (ABH)

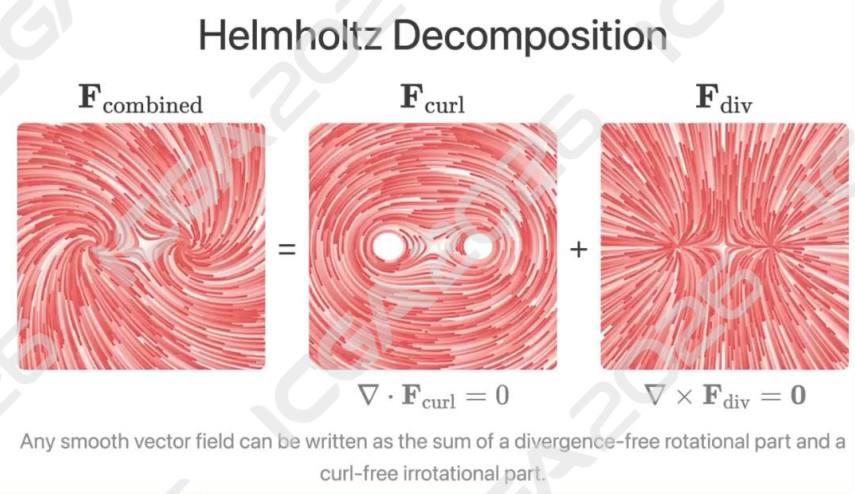
Helmholtz decomposition

$$\mathbf{U} = \nabla\phi + \nabla \times \boldsymbol{\psi}$$

Longitudinal wave

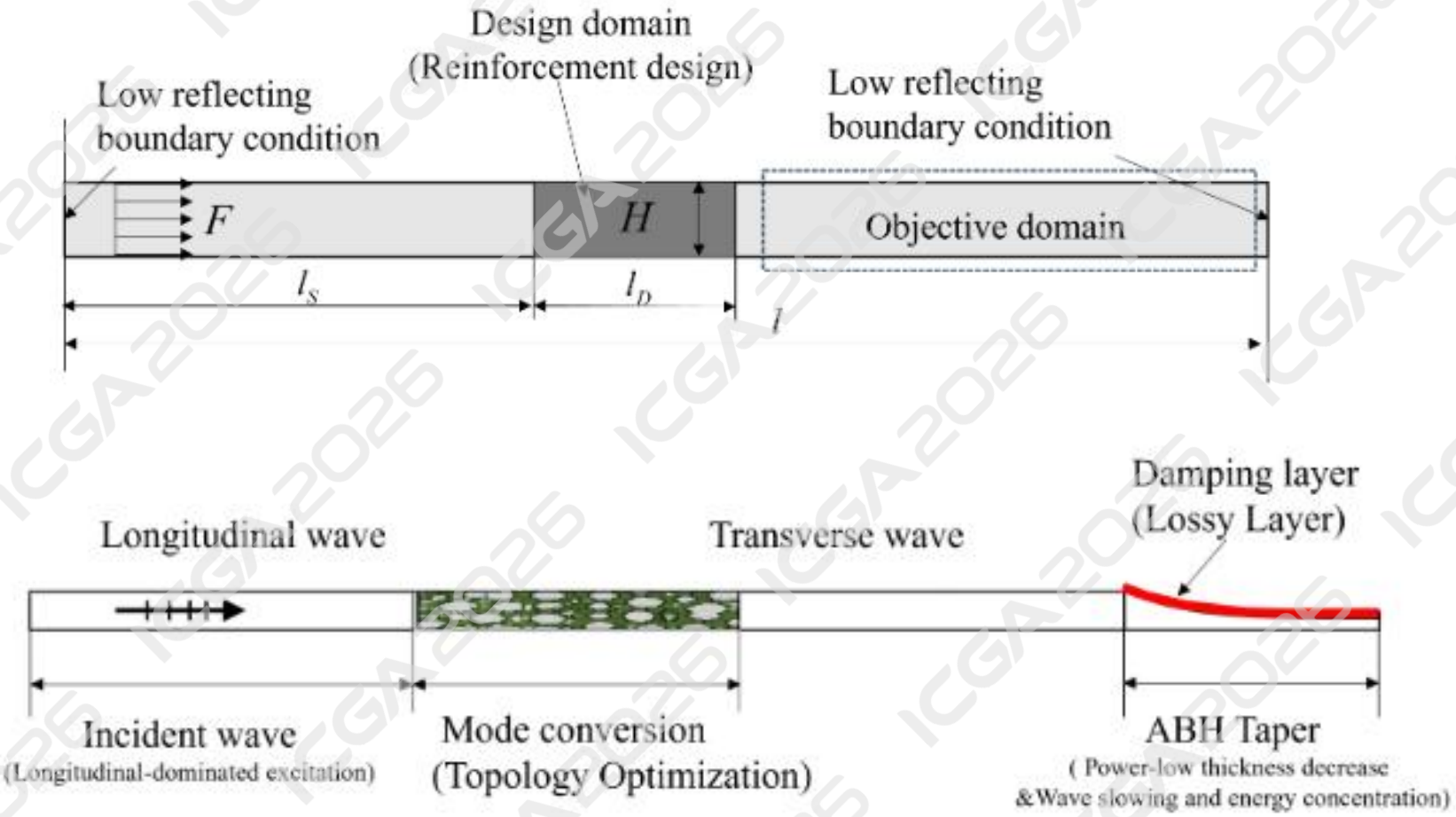
Transverse wave





Alec HelblingAlec Helbling

Mode conversion for acoustic black hole (ABH)



Mode conversion for acoustic black hole (ABH)

Divergence

$$\underset{\gamma}{\text{minimize}} \quad J_{\text{dc}}(\mathrm{U}) = \frac{\int_{\Omega_{\text{out}}} |D|^2 \, d\Omega}{\int_{\Omega_{\text{out}}} |C|^2 \, d\Omega + \delta_{\text{dc}}},$$

Curl

$$\text{subject to} \quad \mathbf{S}(\gamma, \omega)\mathbf{U} = \mathbf{F},$$

$$\mathbf{S}(\gamma, \omega) = \mathbf{K}(\gamma) - \omega^2 \mathbf{M}(\gamma),$$

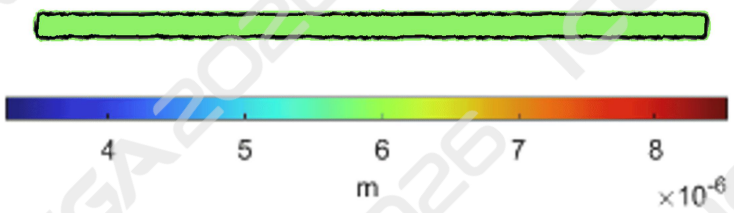
$$\frac{1}{N_e} \sum_{e=1}^{N_e} \gamma_e \leq \bar{\gamma},$$

$$0 \leq \gamma_e \leq 1, \quad e = 1, \ldots, N_e.$$

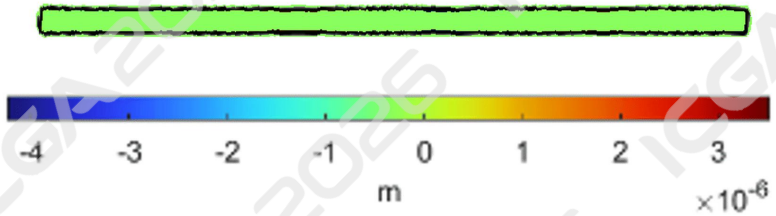
$$\mathbf{U} = \nabla \phi + \nabla \times \boldsymbol{\psi}$$

Mode conversion for acoustic black hole (ABH)

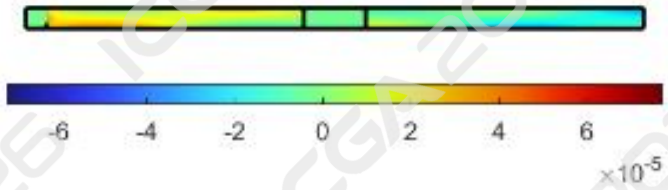




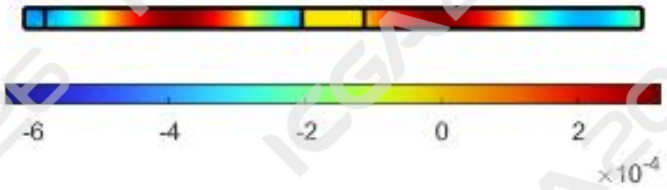
X-Displacement



Y-Displacement



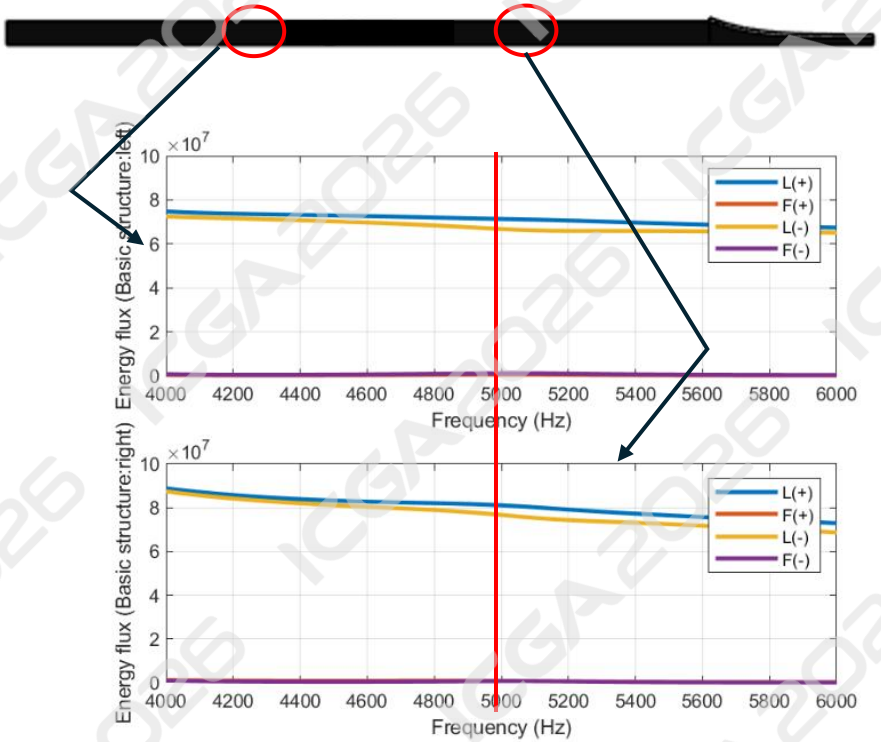
Divergence

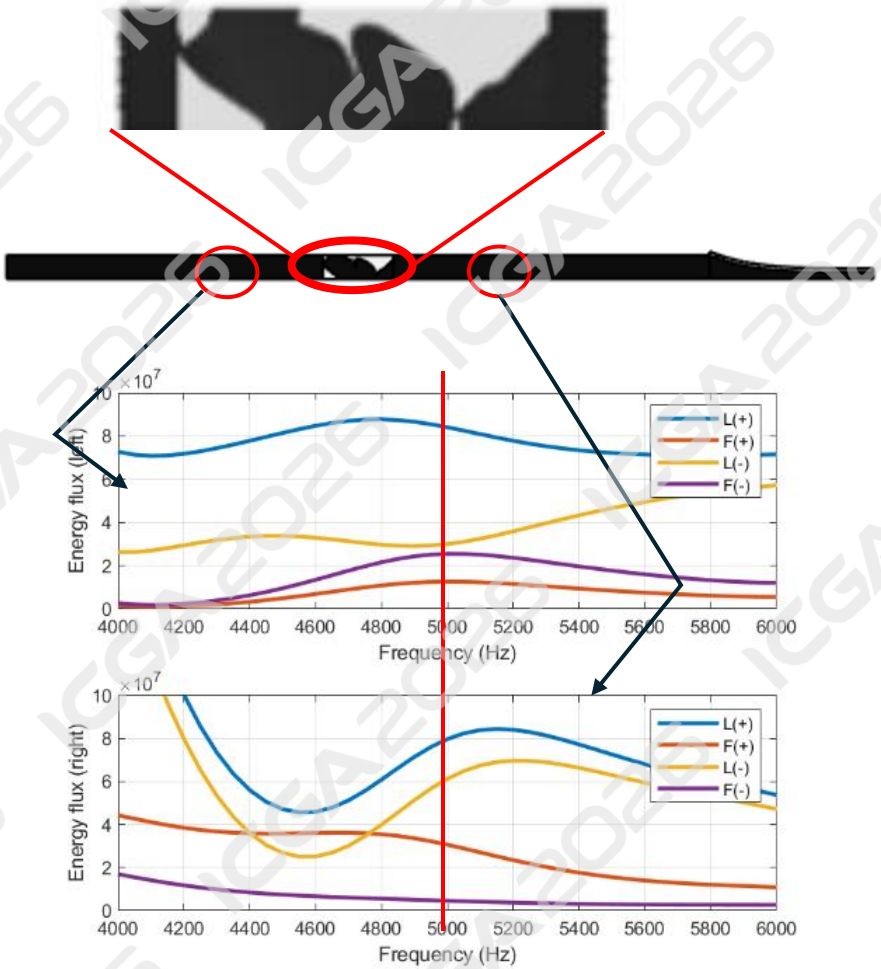


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Mode conversion for acoustic black hole (ABH)

L(+): Longitudinal wave toward right
F(+): Flexural wave toward right
L(-): Longitudinal wave toward left
F(-): Flexural wave toward left





Mode conversion: from longitudinal wave to flexural wave

Mode conversion for acoustic black hole (ABH)

The ratio of the displacement field



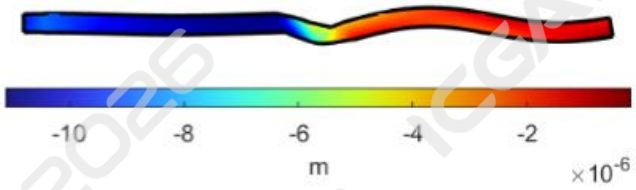
$$\underset{\gamma}{\text{Maximization}} \quad J_{\text{disp}} = \frac{\int_{\Omega_{\text{out}}} |U_y|^2 d\Omega}{\int_{\Omega_{\text{out}}} |U_x|^2 d\Omega + \delta_{\text{disp}}}$$

$$\text{subject to} \quad \mathbf{S}(\boldsymbol{\gamma},\omega)\mathbf{U} = \mathbf{F}, \quad \mathbf{S}(\boldsymbol{\gamma},\omega) = \mathbf{K}(\boldsymbol{\gamma}) - \omega^2\mathbf{M}(\boldsymbol{\gamma})$$

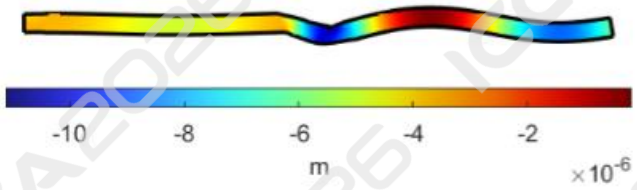
$$\frac{1}{N_e}\sum_{e=1}^{N_e}\gamma_e \le \overline{\gamma}, 0 \le \gamma_e \le 1, e = 1,\ldots,N_e$$

Mode conversion for acoustic black hole (ABH)

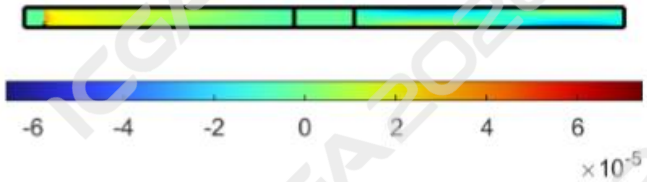




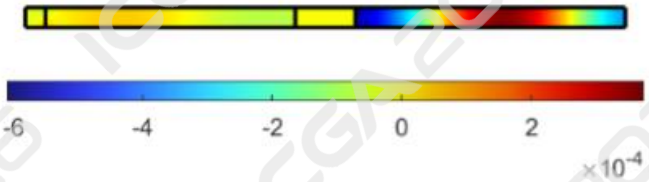
Displacement X



Displacement Y



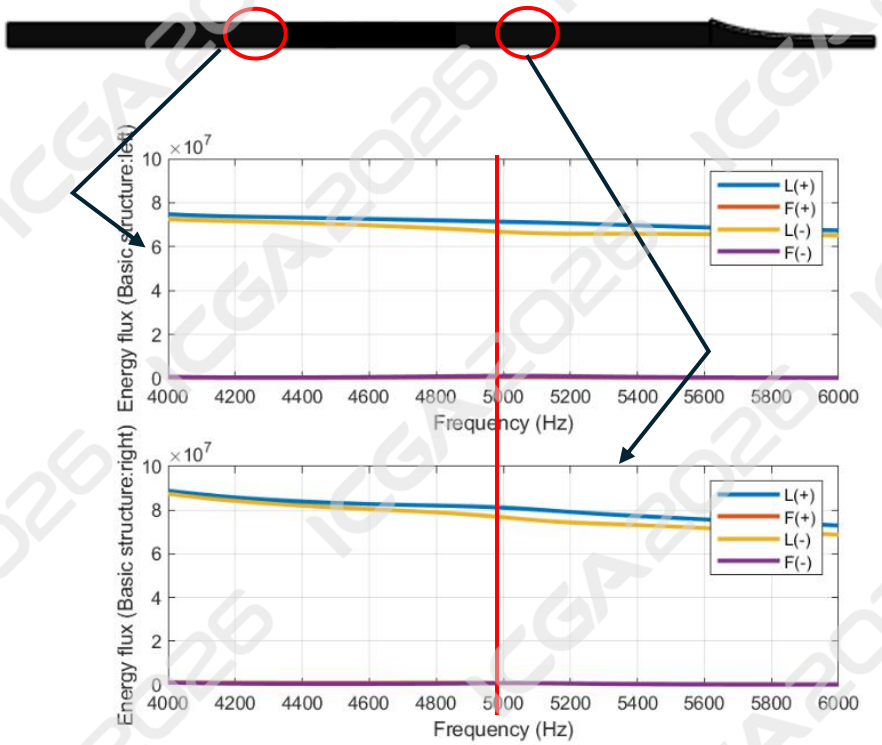
Divergence

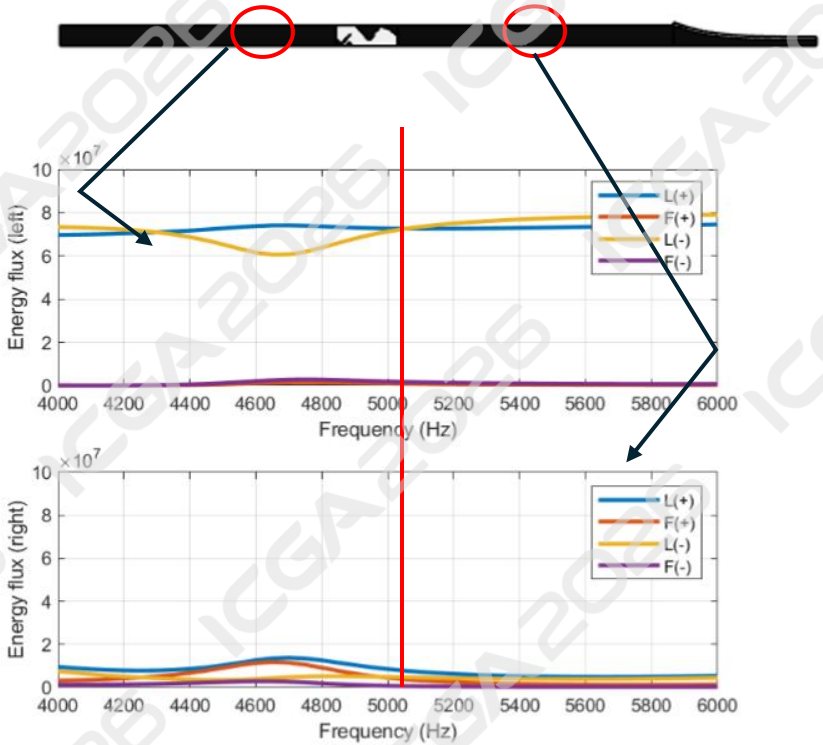


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Mode conversion for acoustic black hole (ABH)

L(+): Longitudinal wave toward right
F(+): Flexural wave toward right
L(-): Longitudinal wave toward left
F(-): Flexural wave toward left





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MultiPhysics System Design laboratory

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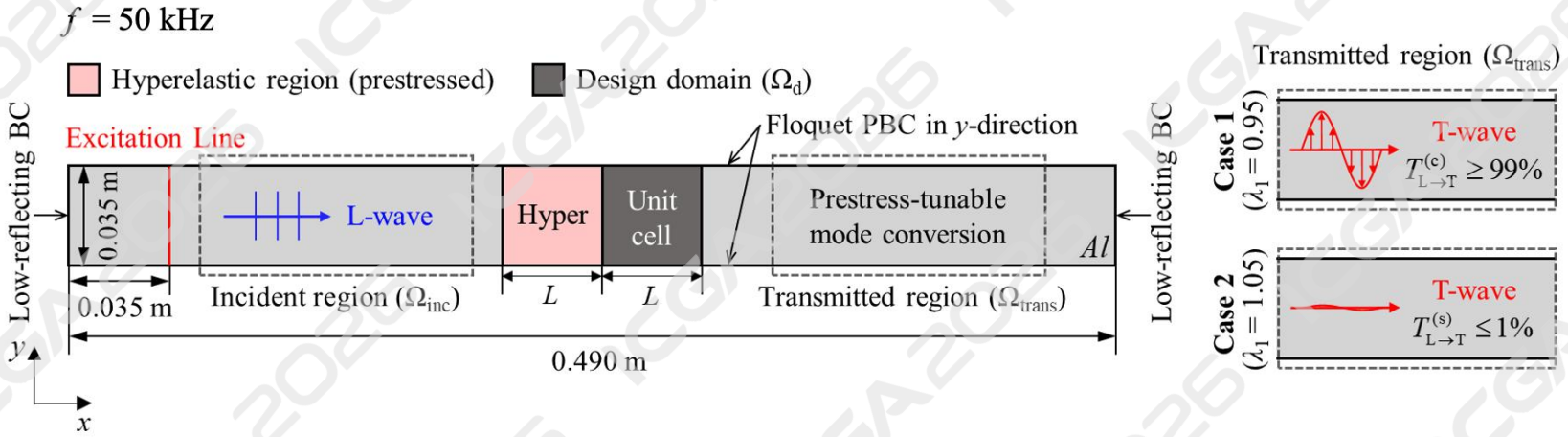
HYU 한양대학교 HANYANG UNIVERSITY

Topology Optimization of Elastic Metamaterials for Prestress-Tunable Wave Mode Conversion Using Hyperelastic Media

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† ghy@hanyang.ac.kr

Problem definition



Physical quantity: Mode converting ratio

Transmission coefficient (L-to-T): $|t_T|^2 = \left|\frac{\text{Amplitude of T-wave}}{\text{Amplitude of L-wave}}\right|^2$

Transmittance (L-to-T): $T_{L\to T} = \frac{\sqrt{\rho_B C^B_{66}}}{\sqrt{\rho_A C^A_{11}}}|t_T|^2$

Switchable

Prestress-tunable mode conversion efficiency ~ 0-100%

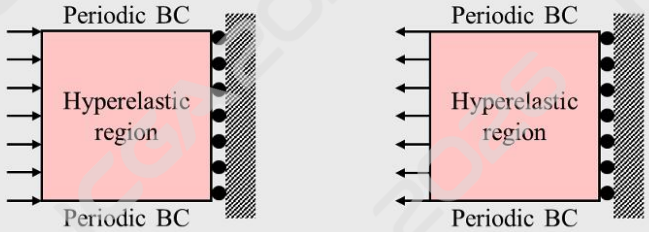
* Hyperelastic material parameters: A_{10} = 80 MPa, A_{01} = 20 MPa, K= 1×10^4 MPa
* C_{11}, C_{66} = Component of elasticity tensor (Base material: aluminum)
* Ω_{inc}, Ω_{trans} = Regions of incident and transmitted wave

Topology Optimization

▪ SIMP



Case 1 Precompressed ($\lambda_1 = 0.95$) **Case 2** Prestretched ($\lambda_1 = 1.05$)



$$\mathbf{K} = \sum \mathbf{K}_e, \quad (e = 1,2,\ldots,NE) \qquad \mathbf{K}_e = \gamma_e^3 \mathbf{K}_{\text{nominal}}$$

$$\mathbf{M} = \sum \mathbf{M}_e, \quad (e = 1,2,\ldots,NE) \qquad \mathbf{M}_e = \gamma_e^3 \mathbf{M}_{\text{nominal}}$$

$\mathbf{K}$: Stiffness matrix — γ_e : Design variable

$\mathbf{M}$: Mass matrix — NE : The number of elements

▪ Optimization problem formulation

$$\underset{\boldsymbol{\gamma}}{\text{Mininize}} \quad J(\boldsymbol{\gamma}) = \frac{1}{NE}\sum_{e=1}^{NE} \gamma_e(1-\gamma_e)$$

subject to

$$C_1(\boldsymbol{\gamma}, f) = \log 10\left(\frac{1-\varepsilon}{T_{\text{L}\to\text{T}}^{(\text{c})}(\boldsymbol{\gamma}, f)}\right) \le 0$$

$$C_2(\boldsymbol{\gamma}, f) = \log 10\left(\frac{T_{\text{L}\to\text{T}}^{(\text{s})}(\boldsymbol{\gamma}, f)}{\varepsilon}\right) \le 0$$

$$\mathbf{SX} = \mathbf{F}, \ \overline{\mathbf{SX}} = \overline{\mathbf{F}}$$

$$\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_{NE}]$$

$$\gamma_{\min} \le \boldsymbol{\gamma} \le 1, \qquad \gamma_{\min} = 1\times 10^{-3}$$

Characteristic

1. Distinct material state
   $$J(\boldsymbol{\gamma}) = \frac{1}{NE}\sum_{e=1}^{NE} \gamma_e(1-\gamma_e)$$
2. Imposition of $T_{\text{L}\to\text{T}} \to 0$ or 1
   Using tolerance ($\varepsilon = 0.01$)
3. Log scale
   For stable optimization
4. Prestress Dependence
   $T_{\text{L}\to\text{T}}^{(\text{c})}$: compressed state
   $T_{\text{L}\to\text{T}}^{(\text{s})}$: prestretched state

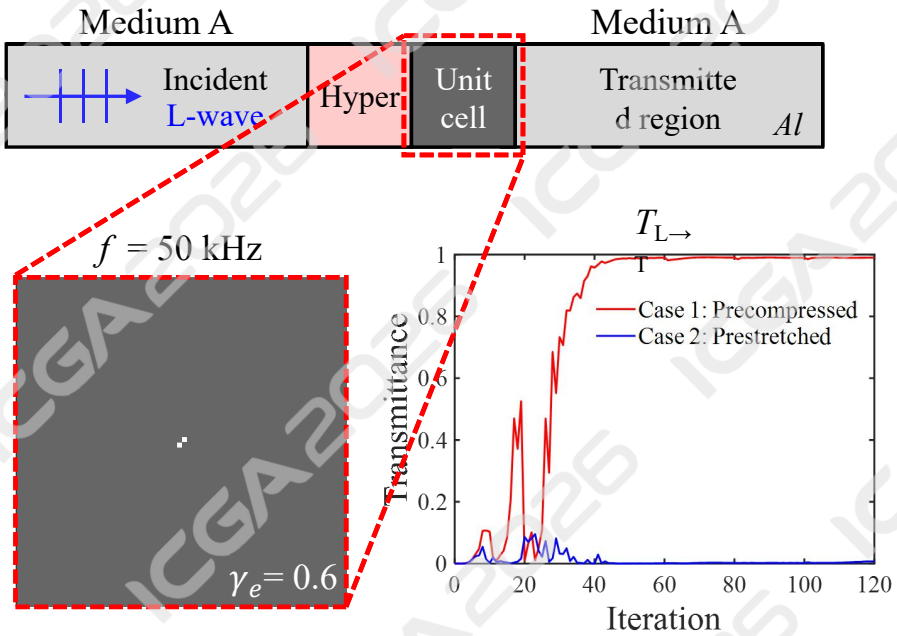
Density filter and *S*-shape function are used.

Weight sum filter: $\Phi(\boldsymbol{\gamma}) = \dfrac{\sum_{e\in Neighbor} \omega_e \times \gamma_e}{\sum_{e\in Neighbor} \omega_e}$ ω_e = weight factor

S-shape function: $\Psi_s(\boldsymbol{\gamma}) = \dfrac{1}{1+e^{-a(\gamma-b)}}$

G.H. Yoon *et al.*, *Structural and Multidisciplinary Optimization*. 2019
G.H. Yoon *et al.*, *KSME international journal*. 2003

Numerical Example

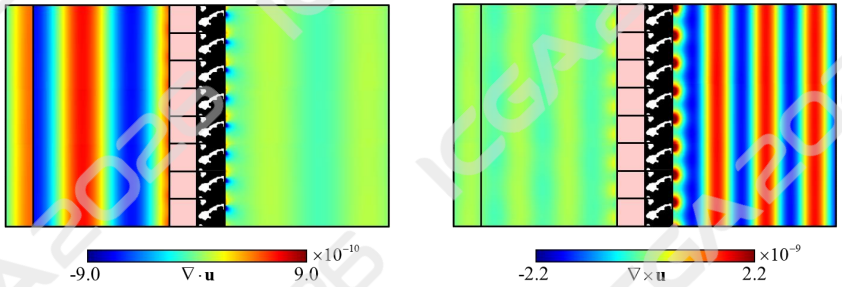


$T^{(c)}_{L\rightarrow T} = 99.01\,\%$ $T^{(s)}_{L\rightarrow T} = 0.42\,\%$

- Certain voids can induce anisotropy in the metamaterials.
- Tunable mode conversion is achieved under different prestress states.

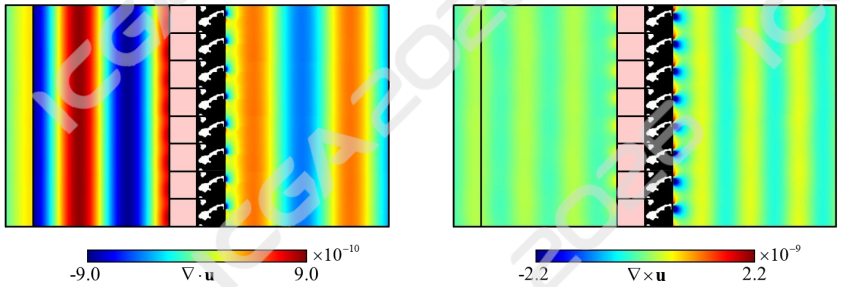
Case 1: Precompressed ($\lambda_1 = 0.95$)

Post-processed (L-wave) Post-processed (T-wave)



Case 2: Prestretched ($\lambda_1 = 1.05$)

Post-processed (L-wave) Post-processed (T-wave)



Conclusions

- This research develops a new monolithic design approach for topology optimization for multiphysics problems (fluid, fluid-structure interaction, two-phase fluid, transient, particle suspended in fluid) and a new optimization method to utilize the ABH for longitudinal wave.

- In order to improve the performance of ABH for longitudinal wave, the Helmtholz decomposition of mechanical wave is studied and based on its decomposition, an optimization formulation considering the divergence and the curl of mechanical wave representing the longitudinal wave and the transverse wave, respectively, is propose.

Thank you

ICGA2026